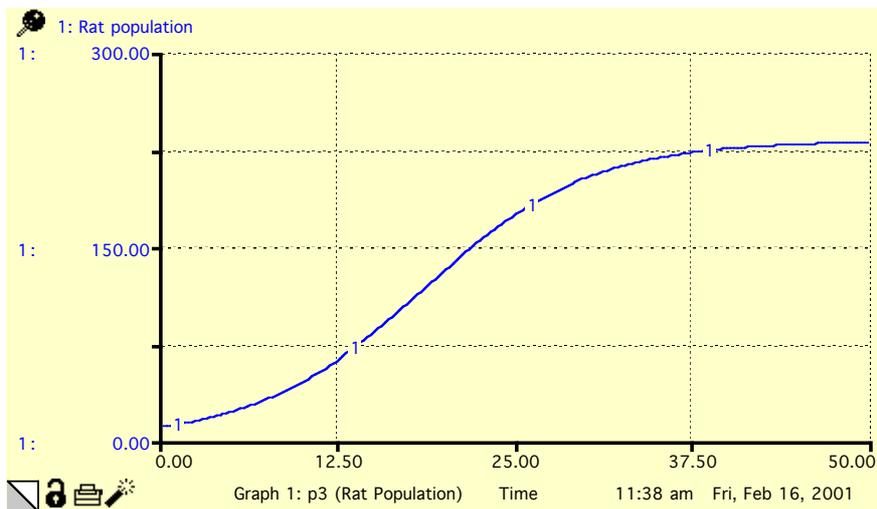


Exploring Models 3

Stella Guide 5

Limits to Growth



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Rat Population

Left to themselves rats breed fast and soon the population grows to fill the environment. Their growth pattern exhibits the classic s-shaped growth as space runs out. To a first approximation it is hoped to explain the limit to growth through infant rat mortality. As population density goes up the survivability of infant rats goes down.

Explore

The relationship between the limit and the different parameters in the model

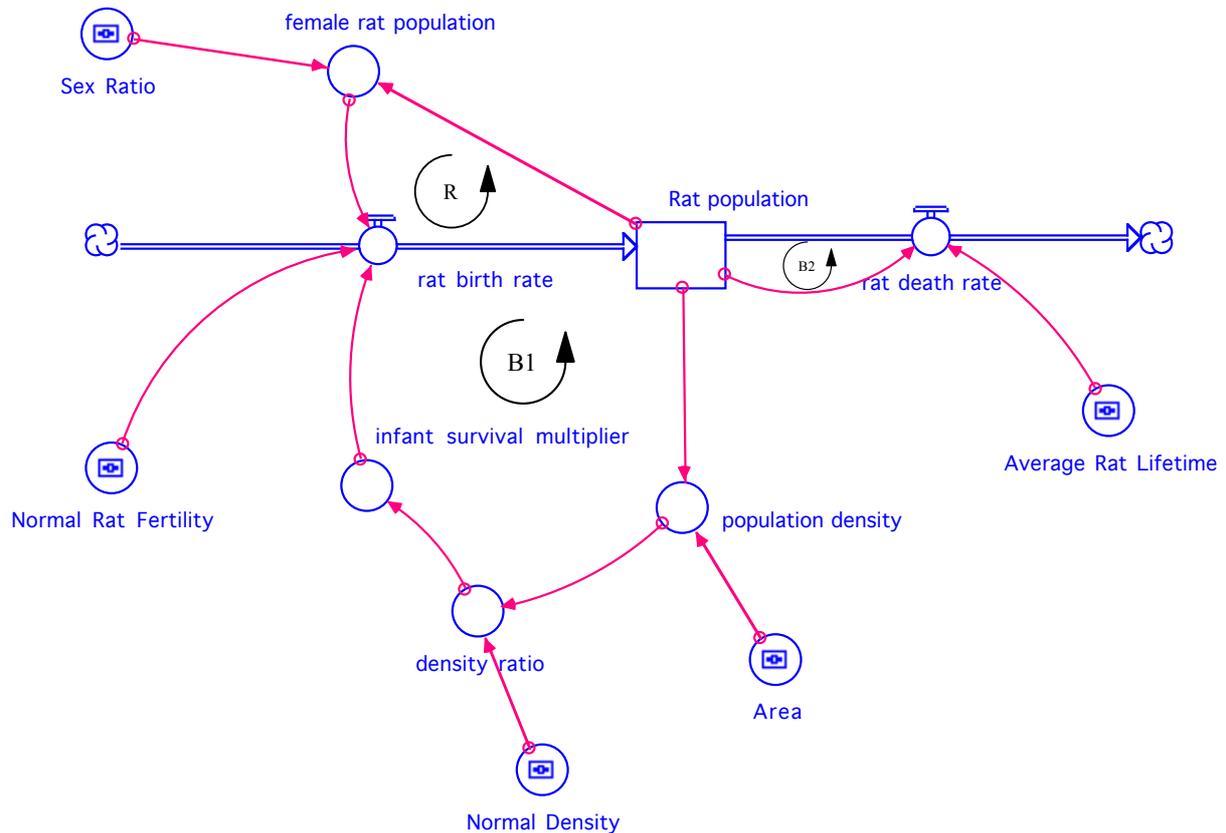
The effect on the growth and the limit of the relationship between infant survivability and population density.

Extend

Extend the model to include increased mortality of adults.

Exercise 1 Rat Fertility and Overcrowding

- *Start up Stella*
- *Place the following limited growth model into the modelling layer of Stella (without the loop identifiers!)*



Essentially the rats have a standard death process. The birth process is the normal one where the birth rate is modified by the population density. As the density increases it exceeds a “normal” density over which fewer infants survive. So infant survival goes down.

- Click on the boxes and set (Note comments are in italics - don't type these!)

INIT Rat_population = 10

rat_birth_rate = female_rat_population*Normal_Rat_Fertility*infant_survival_multiplier
The number of rats born every month

rat_death_rate = Rat_population/Average_Rat_Lifetime
Normal death process - number of rats dying every month

Average_Rat_Lifetime = 32
in months

Area = 11000
Area of the environment(square feet)

population_density = Rat_population/Area
The number of rats per square foot

Normal_Density = .0025
*rats per sq. feet The number of rats per square foot at normal conditions
i.e. when rat fertility = normal rat fertility*

density_ratio = population_density/Normal_Density
The ratio of the current population density to the normal rat density

infant_survival_multiplier = 1-density_ratio/10
*This multiplier is a function of the density ratio.
The higher the density ratio the less infant rats survive
Growth must stop at 10 times the normal density
At present this formula is not quite right!*

Sex_Ratio = .5
The fraction of rats who are female

female_rat_population = Sex_Ratio*Rat_population
The number of rats who are female

Normal_Rat_Fertility = .4
The normal number of rats born per female every month

- *Save your model*
- *Set the run time to 70 and the method to RK4*
- *Confirm the feedback loops on the above diagram by placing in the signs (+ or -) of the connectors/flows to show there is one reinforcing loop R and two balancing loops B1 and B2. You can do this on paper.*

Parameters

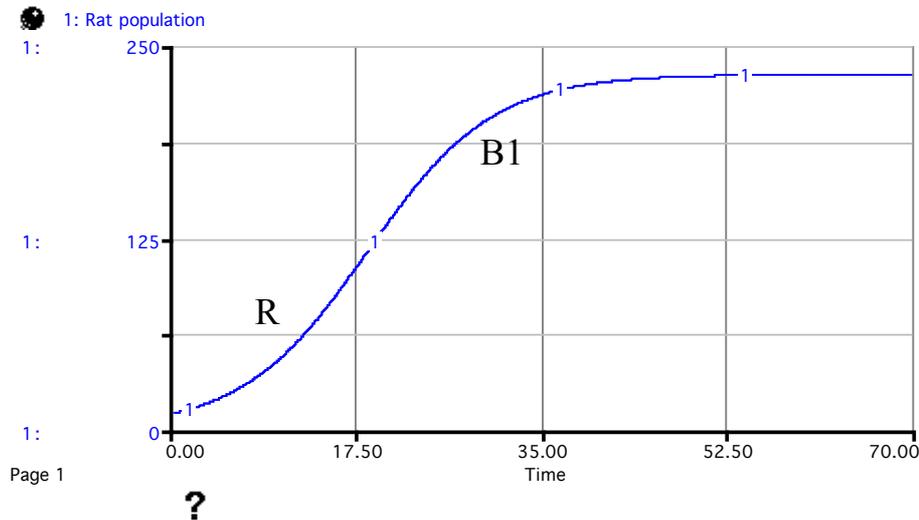
This model appears to have 5 parameter. But fertility, lifetime and fraction female are fairly standard for rats - so the only effective parameters (the ones with leverage) are the area they occupy "Area" and the rather mysterious "Normal Density" - the density at which infant mortality starts

Input and Output Interface

- *Move to the top layer of Stella by clicking the up navigation arrow:*
- *Place in sliders for Normal Density (0 to 0.05) and Area (1000-20000) using the ranges in brackets*
- *Place in a graph for rat population (no axis limits)*
- *On a separate graph place Normal density on the same axes as population density. Infant survival multiplier should also be on this graph (not same axes) The separate graph can be on the same graph pad or a separate one.*

Simulation

Run the model. You should see the population reach a limit of 232. See:



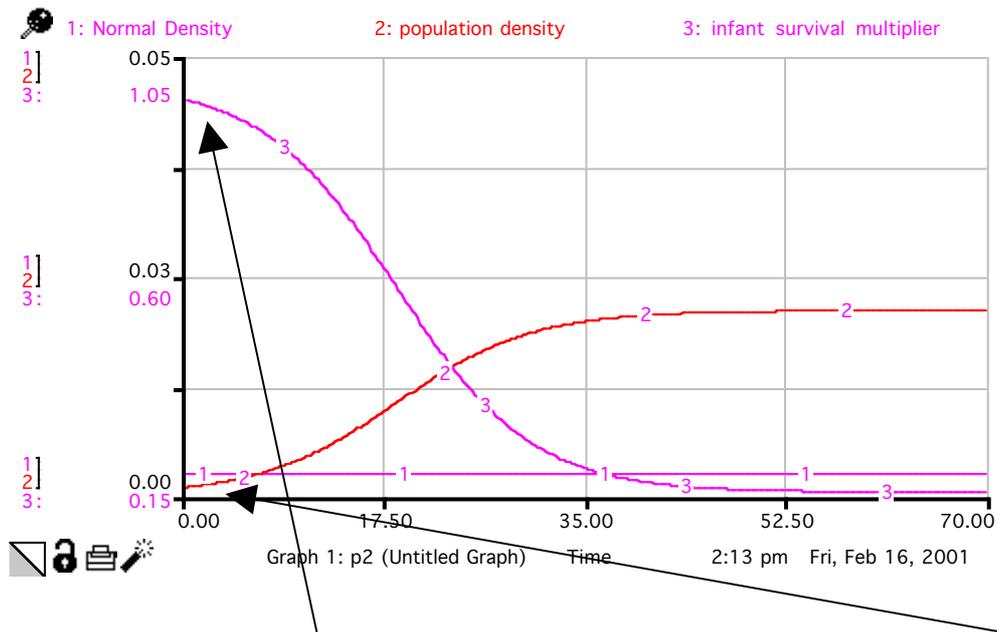
The way the population reaches its limit is S-shaped growth. There is phase where the growth is accelerating in a more or less exponential fashion. This is where the reinforcing loop R is dominant, i.e. it has more effect than the balancing loops. After the inflexion point there is a phase where the balancing loop B1 dominates, slowing the growth to equilibrium.

At any time all the loops have an affect on the actual value of the rat population. The presence of the death process, B2, causes the population to stabilise at a lower value than the carrying capacity determined by the maximum density = $11000 \times 0.0025 \times 10 = 275$.

N.B. The limit to growth is the stable equilibrium point of the system. You can calculate this by setting inflow = outflow. Do this exercise outside your lab. It is the equivalent of finding the equilibrium points of the differential equation.

The issues of loop dominance are outside the scope of this introductory lab. However it should be noted that there is a brief period after the inflexion point where both B1 and B2 are needed to dominate R and thus initiate the slowing down of the population growth.

You should also see the other graph as:



Note that infant survival is dropping even when the population density is under the normal density. This is wrong there should be no effect on population density until normal density is exceeded.

- *Change Area and Normal Density and check that they affect the sustainable population limit in the correct way*

Exercise 2 - Fixing Infant Mortality Under the Normal Density

Use “If Then Else”

Infant survival should be 1, meaning maximum survival, as long as the population density is under the normal population density. That is as long as density ratio is less than 1.

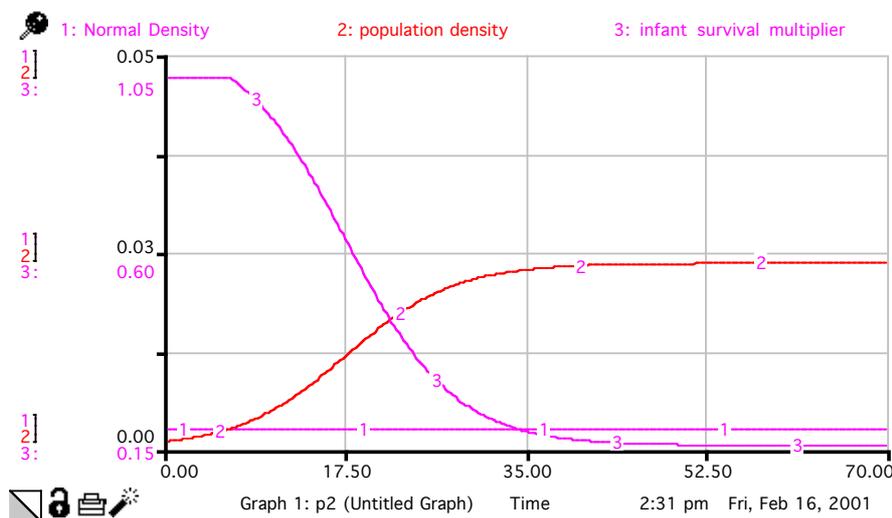
- Thus set the formula in Infant Survival Multiplier to

IF (density_ratio <1) THEN(1) ELSE (1.1-density_ratio/10)

Note the “else” formula has been changed so that it is 1 at density ratio = 1 preserving continuity

You can select the if then else from the function menu if you prefer - use upper case letters if you type them.

Now re-run the model with the default level of parameters and the densities should be:



There is little change in the upper limit of the population. Growth is still s-shaped. Although the model in exercise 1 has been improved it was still sufficient to explain why growth was limited in terms of the reinforcing and balancing loops. What exercise 2 has achieved is a refinement in the fine detail of the model.

Handling situations like this by differential equation is quite hard because you have to solve it for both parts of the if and then “match” the solutions together! Computer simulation however is much easier providing you check the solution remains intact for a variety for times steps (DT), as a discontinuity in the slope of a variable, is in curve 3 in the graph above, can induce numerical inaccuracies.

Use of Graphical Relationship

An alternative way to define the relationship between two variables is to assign the relationship graphically. This is particularly useful if the relationship is only known experimentally. In the case of density and Infant survival there is no formal only data for survivability at different densities.

Set up a graphical relationship for Infant Survival Multiplier as follows:

- *Double click* Infant Survival Multiplier and delete its current formula
- Simply *place* density ratio in the formula box and *click* become graph
- A graph menu has appeared. You can either set the graph by clicking the mouse, or by setting the table as given below. Note the horizontal scale should be 0 to 10 and the vertical scale 0 to 1.

Input	Output
0	1.000
1	1.000
2	0.960
3	0.920
4	0.820
5	0.700
6	0.520
7	0.340
8	0.200
9	0.140
10	0.100

Note that the function remains at 1 when density ratio is between 0 and 1 - as required.

Go back to the interface layer and run the simulation. The result looks quite similar to before.

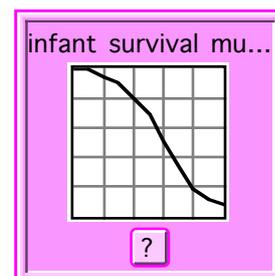
Now set up input for the graph.

- Select the graphical input icon



This will give the graphical input device

- *Double click* at set to infant survival as usual. Click OK.



When you want to make changes to the graphical relationship just double click this graph input pad. It is easier to make changes with the mouse than the table.

As an exercise try a few different graphical relationships - as long as the curve is monotonically decreasing that is fine. Infant survival must go down as density goes up.

Note that little changes in the result.

Equilibrium Point

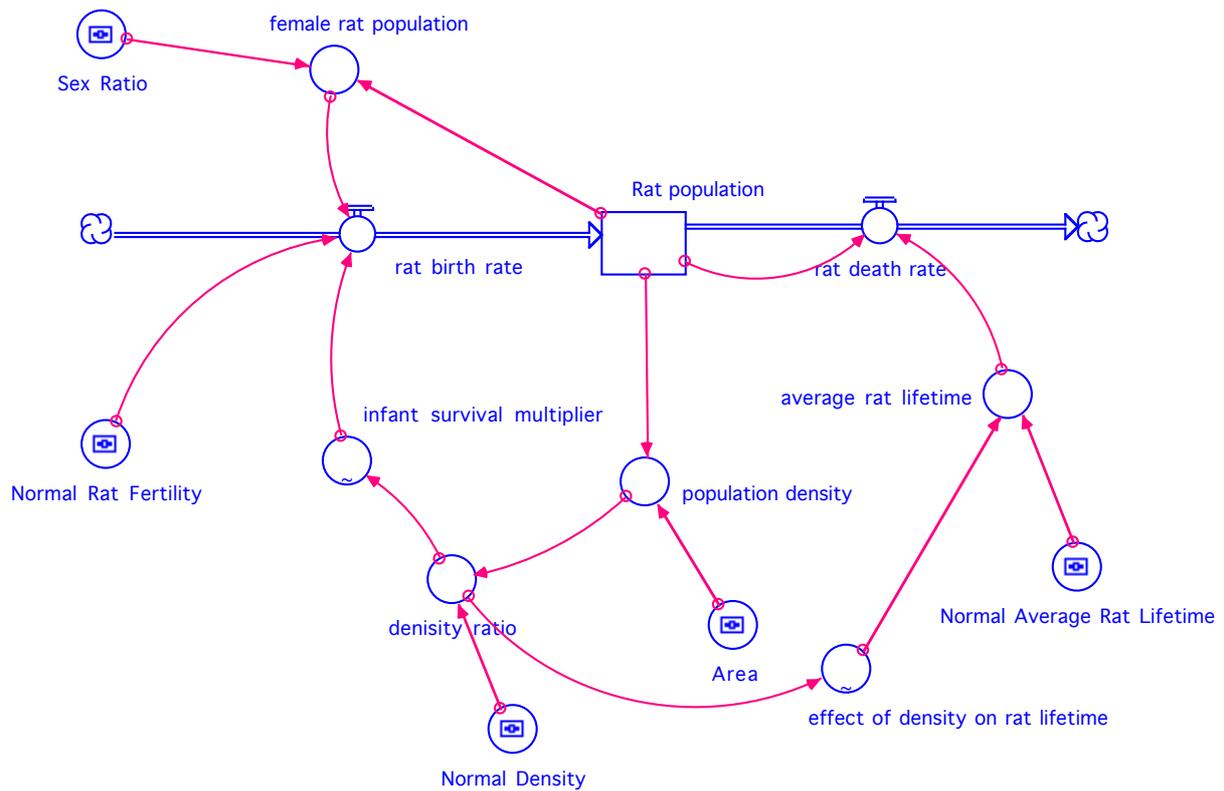
By the way, the limits to growth model is an other example of a PATH INDEPENDENCE. Try some different initial values and it always ends up at the same maximum sustainable value. That maximum sustainable value is an example of a STABLE EQUILIBRIUM point.

Set the initial value of the rat population to be above the equilibrium value. You should be able to show that the population falls to equilibrium. The point is stable from both sides.

Exercise 3 Adult Mortality - Open Exercise

The following is an extended model where the deaths in adult rat population are also affected by crowding effects.

Extend your model using the following outline. Fill in the relationships in the new connectors and examine the effects on the rat population.



You should be able to show the equilibrium population is reduced further, but the behaviour is still S-shaped growth. If the model were used to illustrate S-shaped growth due to carrying capacity there would be no need to include crowding on both births and deaths. One is sufficient to demonstrate the principle. Only if the model was being fine-tuned would you insist on both effects being included.