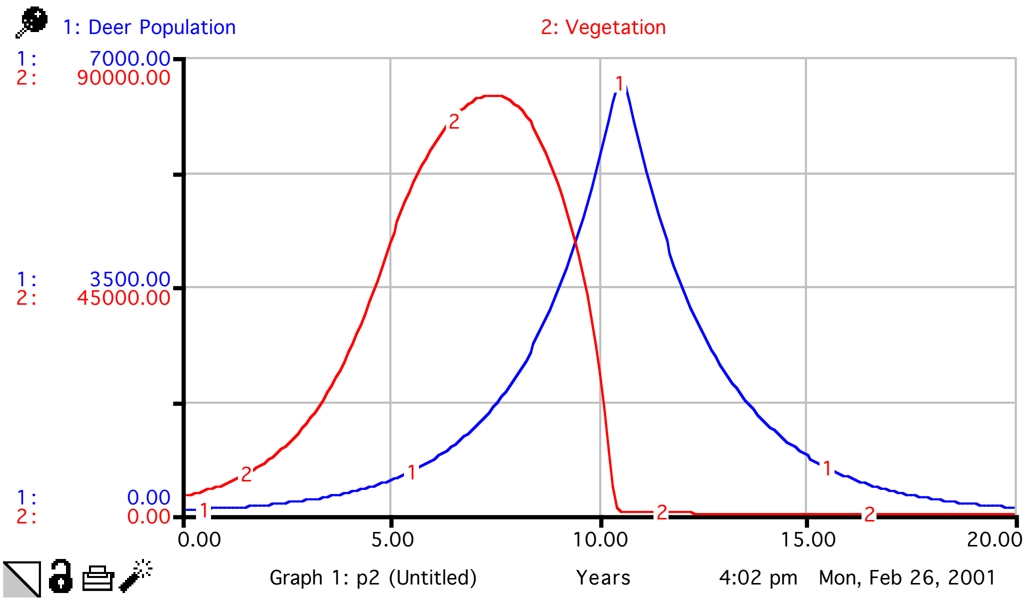
Building System Dynamics Models

Stella Guide 6

Overshoot and Collapse



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Deer and Vegetation

Deer live on a plateau with sufficient vegetation to sustain them. Being a plateau it is difficult for them to leave or for new herds to join them. Being a plateau their food supply, the vegetation, is limited as it can only cover a fixed area.

Can the deer population survive?

Build

You will be asked to build up the model step by step using the numerical information provided.

Exercise 1 Births and Deaths of Deer

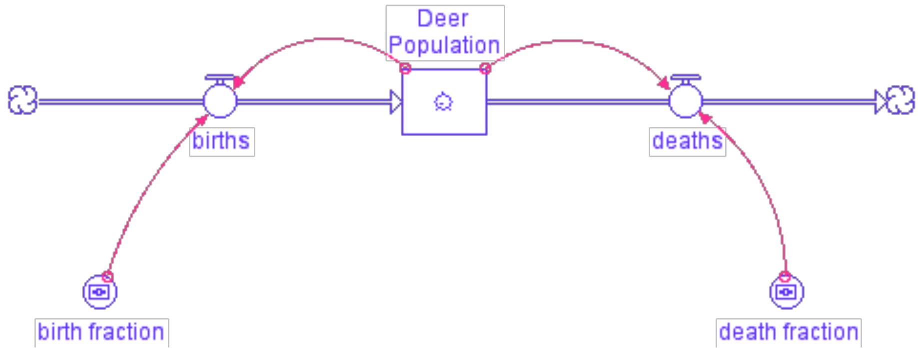
A species of deer live on an isolated plateau and reproduce each year. Each female of childbearing age produces an average of 1.6 young each year, i.e. twins are born about every other year. Only 2/3 of the female population are of childbearing age and females comprise about half the deer population.

- *What is the net birth fraction (per capita rate)?* ....

Deer live for an average of fifteen years.

- *What is a suitable estimate for the net death fraction (rate)?* ....

Produce in Stella a simple birth death model for this situation, given you start with 100 deer. Place an output graph and controls for the birth fraction and initial deer population on the top level



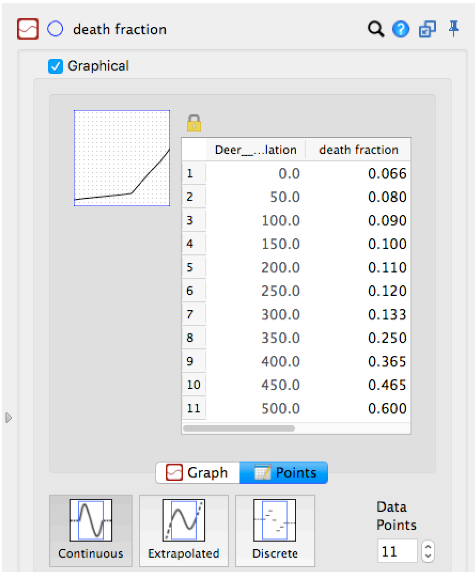
In run specs set the end time to 20 years with RK4, step length 0.1. Set the time unit to years.

- *Describe the behaviour of the deer population.* ....

Exercise 2 Limited Food

The deer live off the vegetation on the plateau. As long as the deer population is less than 100 the food supply is no problem and the only deaths are natural ones. However it has been observed that in excess of 100 the death rate is higher. For a population of 300 the death rate is running about double normal. Once when the population hit 500 (due to random effects in deer and vegetation growth), the death rate due to starvation effects was 60%.

Construct a suitable limits to growth model, given death fraction is now a graphical converter (covered in Stella Guide 5). You can use the diagram below as a guide or mark in the three known points at deer population values of 100, 300 and 500, and then join the points. The graph does not need to be exact



	Deer_...lation	death fraction
1	0.0	0.066
2	50.0	0.080
3	100.0	0.090
4	150.0	0.100
5	200.0	0.110
6	250.0	0.120
7	300.0	0.133
8	350.0	0.250
9	400.0	0.365
10	450.0	0.465
11	500.0	0.600

- *Determine the maximum sustainable deer population. . I.e. the equilibrium value. ………..*
- *Identify the loops in the above model and relate them to the graph of vegetation.*
- *What happens if the deer population starts above this equilibrium value?*

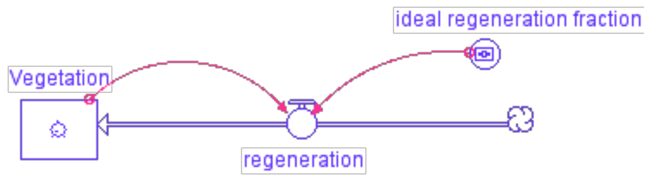
....

Exercise 3 Vegetation Model

a) The model in exercise 2 is unsuitable because there is no model of the vegetation growth.

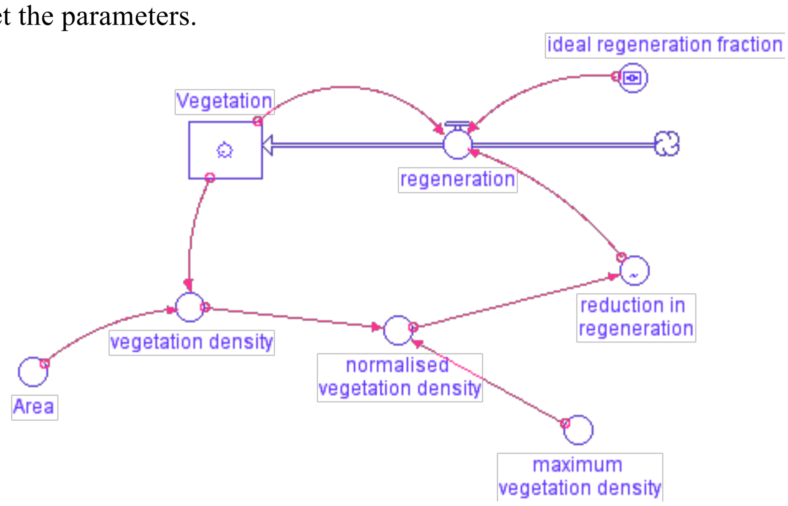
The vegetation grows rapidly and spreads at a rate of 75% each year, due to the net effect of seeding, growth and decay.

There is initially 3,500 biomass of vegetation. Produce a model of the vegetation in the same Stella file as the deer model. Add controls for *Vegetation* and *ideal regeneration fraction*, and add *Vegetation* to the graph of the deer. Run the model and examine the graph of the vegetation. It should be exponential.



b) Clearly there must be a limit to the growth of the vegetation. The plateau has an area of 1000 hectares and vegetation can grow to a maximum density of 100 biomass per hectare.

Extend your vegetation model to include this limit to its growth. Vegetation density is in units of biomass per hectare. The units are the key to the formulae! The numbers in the previous paragraph set the parameters.



The model above converts the vegetation density into a normalised vegetation density. This is unitless and on a scale from 0 to 1. This modelling device makes it easier to handle the effect of the vegetation density on its own growth rate through *reduction in regeneration*, also on a scale 0 to 1.

reduction in regeneration is made a graphical converter. It can be ANY monotonically decreasing function from (0,1) to (1,0). (Monotonically decreasing means it is always decreasing; there are no periods of increase. See solutions at the back for an example.) Reduction in regeneration multiplies the ideal fraction thus reducing regeneration as vegetation increases.

- *What is the maximum sustainable limit of the vegetation growth in your model (in biomass)? Does it agree with the numbers above?*
- *Identify the loops in the above model and relate them to the graph of vegetation.*

Notice how the use of unitary scales, i.e. 0..1, makes model construction easier.

Exercise 4 Combining the Deer and Vegetation

Of course the deer and the vegetation are ONE system with each affected by the other. The two models will be combined in two stages:

a) The Effect of the Deer on the Vegetation

The deer require 15 biomass units of vegetation per year to live.

Extend the vegetation model of exercise3 so that the vegetation is reduced due to consumption by the deer.

Run the model again starting with 100 deer. You should find the vegetation quickly dies out.

- *Are there starting levels of the deer population that cause the vegetation to survive?*

  ....

- *Are there starting levels of the deer population that cause the vegetation to survive?*

  ....

You should check your model with the solutions at the back before proceeding.

b) The Effect of the Vegetation on the Deer

Of course the deer's death rate will now be affected if there is insufficient food. The deer's act of consuming the vegetation feeds back on its own death rate. There is now a new feedback loop!

Delete the link from deer to the death fraction. Instead the death fraction will depend of the amount of vegetation. If there is no vegetation the death rate is 100%. However it has been observed that once the vegetation reaches 1000 biomass at any time (a coverage of 1% of the plateau) the death rate is the normal value of 1/15 = 0.067. For coverage of less than this the death rate increases as food becomes difficult to find on a daily basis.

Extend the model to include the above effect. (Hint: keep death fraction as a graphical converter, now from vegetation, and have the vegetation axis from 0 to 1000)

Run the model with 50 initial deer and 3500 initial vegetation.

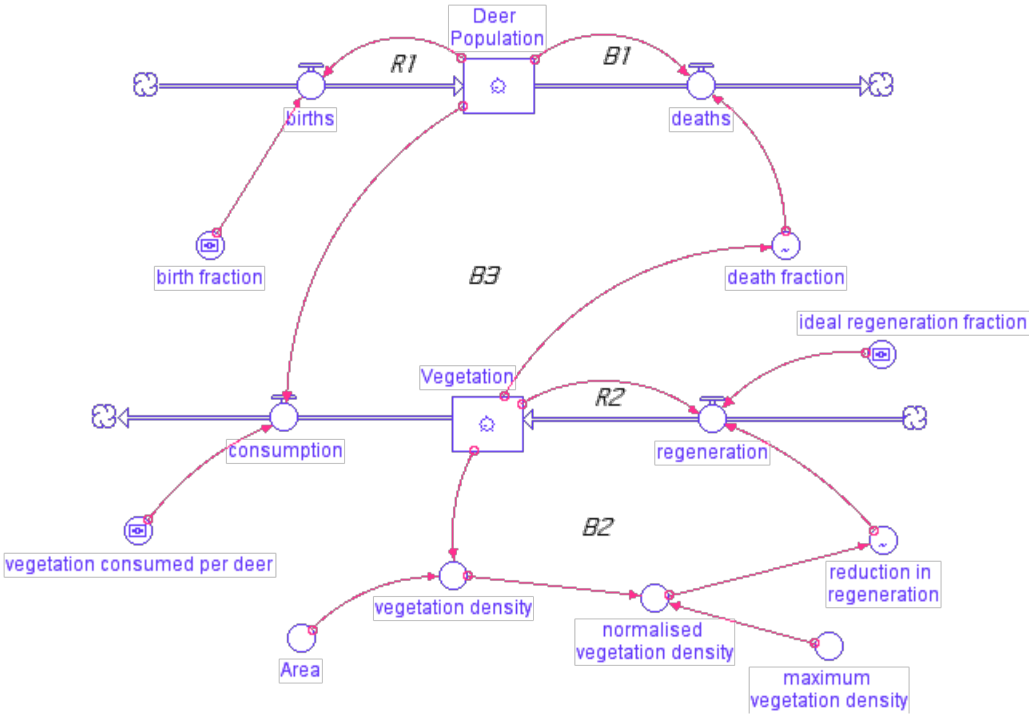
You should find the population grows then collapses - called overshoot and collapse

- *How many years before the vegetation reaches its peak?* ....
- *How many more years before it is wiped out?* ....
- *What sort of advanced warning was there of impending doom?* ....
- *What was the vegetation's maximum value?* . ...
- *When did the deer population start going downs?* ....

Check the solutions at the back before proceeding.

Feedback

Your model should now include a new loop between the deer and the vegetation. The loops have been given labels:



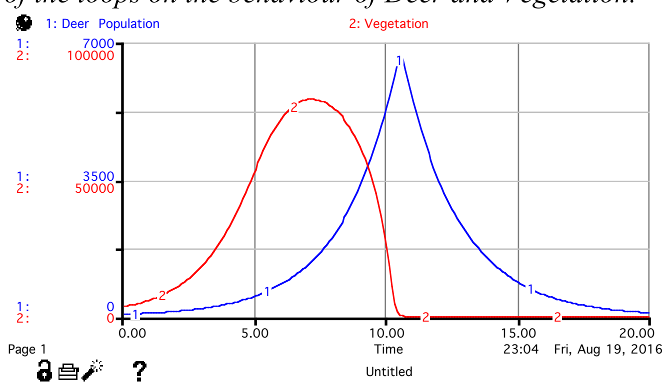
Up to now all the feedback loops you have seen have been **first order**. That is they involve only one stock linked to itself, either directly, or through one or more converters. However the new loop, *B3*, involves two stocks; it is called **second order**.

- *By identifying link polarities <u>show that</u> the second order loop in the deer vegetation model is balancing.*

Identifying the effects of first order loops on the graph of a stock is fairly straightforward. First order reinforcing loops accelerate growth or decline, first order balancing loops slow down growth, or decline. However second and higher order loops are much harder to identify in stock behaviour, especially if each stock in the loop is subject to other loops.

However a useful guide is that second (and higher) order balancing loops are often responsible for a stock changing from growth to decline (or decline to growth). That is they are able to flip the direction of travel of a stock.

- *<u>Identify</u> the effects of the loops on the behaviour of Deer and Vegetation.*



Note:

- *B3* may dominate on vegetation in different periods to its dominance on the deer.
- It takes advanced tools to precisely identify which loop dominates at any given point.
- There are different definitions of "dominance". It may be which loop has the greatest effect. But it could also be which loop, or combination of loops, is responsible for the way the graph curves.

Thus there is no uniquely right or wrong answer.

See the solutions before continuing.

Extended Exercise - Avoiding Catastrophe

If you think about it there are not many parameters in this model you can change.

- *Which parameters do you think you have some control over?* ....
- *If so change the values to see if you can get the deer and the vegetation to survive.*

Changing a parameter is called **leverage**. The trouble with ecological problems like this is there is often very little leverage.

There are normally only two ways to improve a system, or find a solution to a system with a problem:

1. Leverage - change parameters values
2. Structural - Add in extra links (as you did in staff recruitment - the drifting goal), or flows or in extreme cases extra stocks! That is make new policy decisions.

Here are some suggestions involving extra flows and maybe some links that may help the deer survive:

Culling Deer	(flow out of deer)
Reintroducing deer	(flow into deer)
Reintroducing vegetation	(flow into vegetation)

- *Try one or more of the above three solutions with constant flows - can you establish survival?*

Of course you could let your rate of culling, reintroduction etc. depend on other stocks, flows or converters in the system, that is use feedback to control culling or reintroduction.

- *Try one of the above solutions with such a variable flow. Remember if you link from a rate (flows and some converters) you must smooth because they are instantaneous (see Stella Guide 4). Can you achieve survival?*

Ultimately the problem is a lack of predator. Without a predator deer need to be farmed, with fencing to control reproduction. Additional food may be required.

Failings of the Model

- The deer decline exponentially, thus there are still many deer left even when there is no food. The effect of zero vegetation on death rate needs improving.
- The effect of the vegetation on the deer death rate was very specific to this situation. Would deer death rate start increasing if there were less than 1000 biomass of vegetation, or if the plateau were bigger and could sustain more vegetation? Should it not be vegetation density that links to the deer death rate? Does not the density of deer matter, given they are geographical spread over the area? These are issues that are difficult to resolve completely.
- As food gets scarce deer may have to spend time searching for food, especially if the deer are not homogeneously spread across the plateau. Migration around the plateau could allow vegetation patches to recover.
- Do young deer have less reliance to food shortage than adults? If so a cohort model may be more appropriate, allowing the effective birth rate to fall when food is scarce.

However the model is sufficient to demonstrate the principle of over shoot and collapse of resource consumption in a finite area. The model has been applied to human consumption of resources in the world, with similar results.

Solutions

Exercise 1

- *What is the net birth fraction (per capita rate)?   1.6/2*(2/3) = 0.533   per year*

This assumes the fraction of population female and the fraction of females who give birth remain constant.

You can put the formula in the equation for the converter

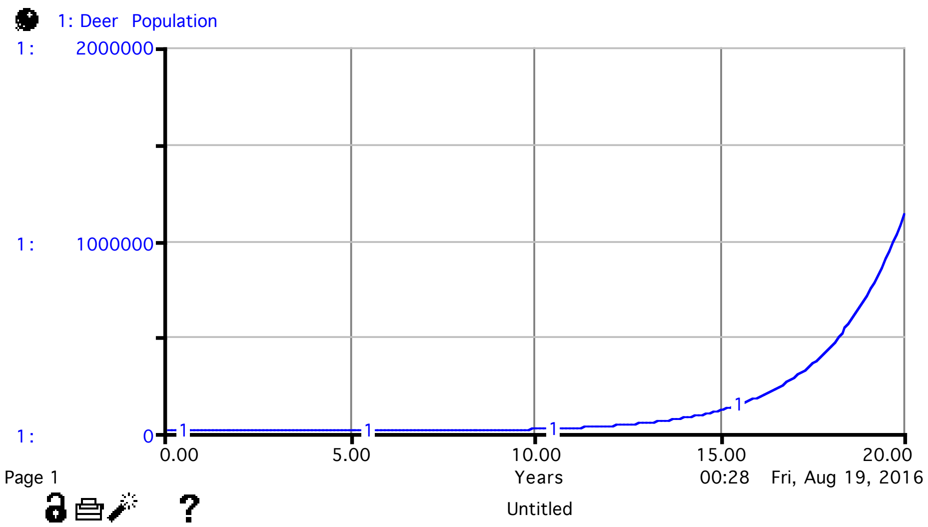
- *What is a suitable estimate for the net death fraction (rate)?   1/15 =0.066 per year*

This is only an estimate as death rate depends on an age distribution in equilibrium, not just lifespan.

The equations are:

Deer__Population(t) = Deer__Population(t - dt) + (births - deaths) * dt
INIT Deer__Population = 100
INFLOWS:
births = birth_fraction*Deer__Population
OUTFLOWS:
deaths = Deer__Population*death_fraction
birth_fraction = 1.6/2*(2/3)
death_fraction = 1/15

The behaviour is exponential with huge growth: in 20 years to over 100,000 deer.



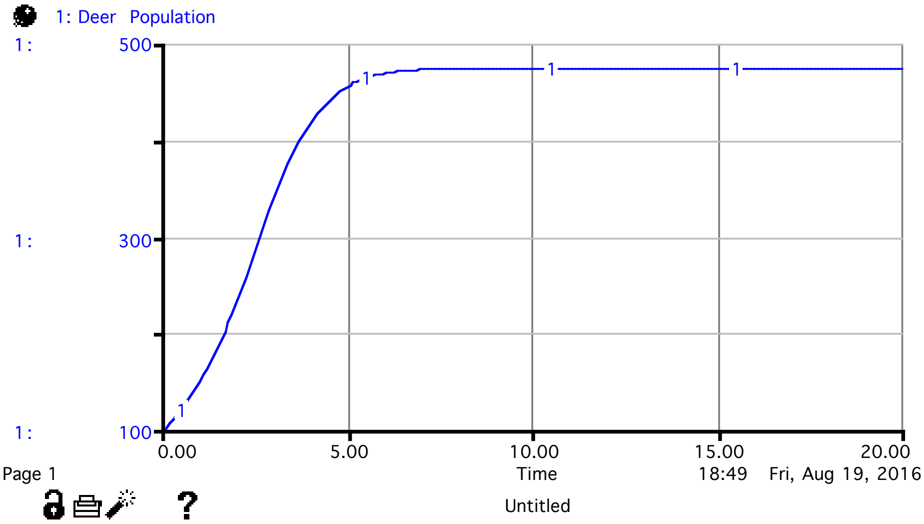
From the SD diagram on page 3 the birth loop is reinforcing and the death loop is balancing. The birth fraction exceeds the death fraction, thus the reinforcing loop dominates, giving exponential growth. See also Stella guide 1.

Exercise 2

Equations:

Deer__Population(t) = Deer__Population(t - dt) + (births - deaths) * dt
INIT Deer__Population = 100
INFLOWS:
births = birth_fraction*Deer__Population
OUTFLOWS:
deaths = Deer__Population*death_fraction
birth_fraction = 1.6/2*(2/3)
death_fraction = GRAPH(Deer__Population)
(0.00, 0.067), (50.0, 0.08), (100, 0.09), (150, 0.1), (200, 0.11), (250, 0.12), (300, 0.133), (350, 0.25), (400, 0.365), (450, 0.465), (500, 0.6)

Result is S-shaped growth, see Stella Guide 5.



- *Determine the maximum sustainable deer population. I.e. the equilibrium value.*

  475 depending on the graphical converter for death fraction.

- *<u>Identify</u> the loops in the above model and relate them to the graph of vegetation*

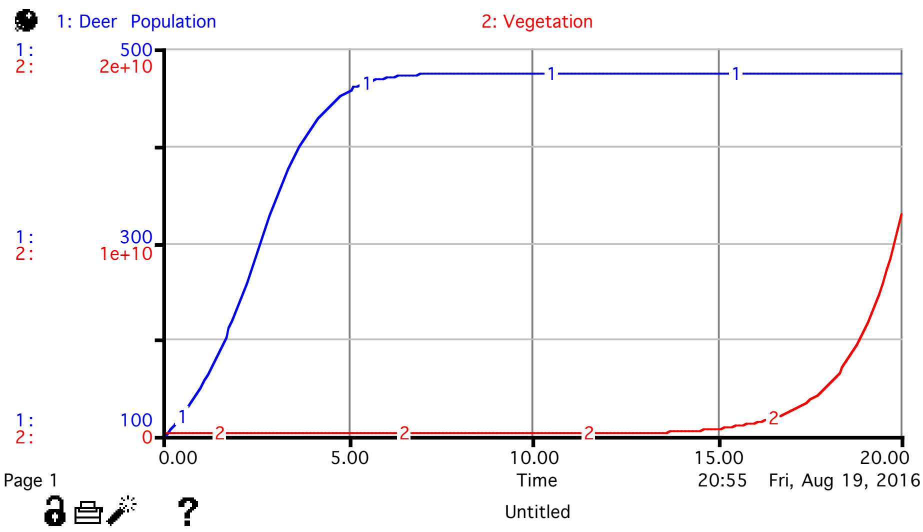
  The loop through births is reinforcing. It causes the initial acceleration in deer population. The direct loop through deaths, and the one via death fraction are balancing. The latter slows down the increase in deer population until the death fraction matches the birth fraction, bringing growth to a halt. See Stella Guide 5.

- *What happens if the deer population starts above this equilibrium value?*

  The population falls to this value

Exercise 3

a)

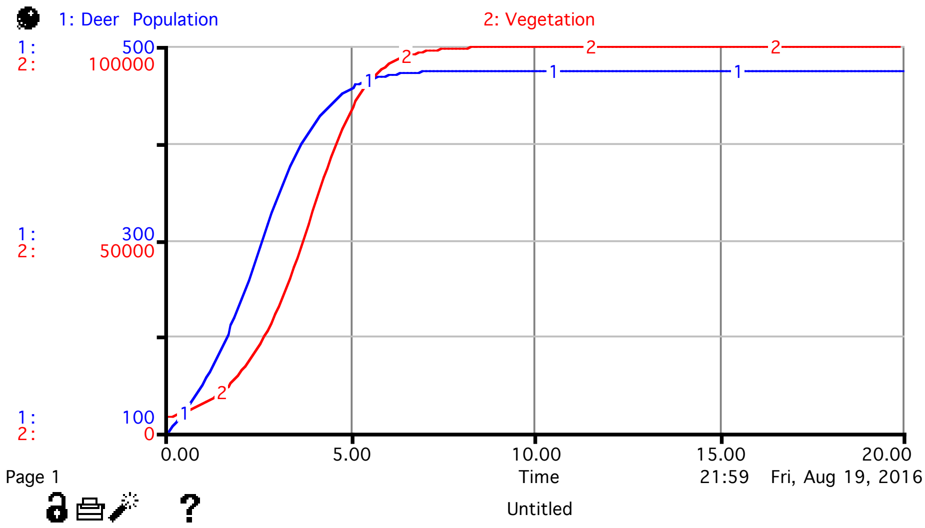


b)

Deer__Population(t) = Deer__Population(t - dt) + (births - deaths) * dt
INIT Deer__Population = 100
INFLOWS:
births = birth_fraction*Deer__Population
OUTFLOWS:
deaths = Deer__Population*death_fraction
Vegetation(t) = Vegetation(t - dt) + (regeneration) * dt
INIT Vegetation = 3500
INFLOWS:
regeneration = ideal_regeneration_fraction*Vegetation*reduction_in__regeneration
Area = 1000
birth_fraction = 1.6/2*(2/3)
death_fraction = GRAPH(Deer__Population)
(0.00, 0.066), (50.0, 0.08), (100, 0.09), (150, 0.1), (200, 0.11), (250, 0.12), (300, 0.133), (350, 0.25), (400, 0.365), (450, 0.465), (500, 0.6)
ideal_regeneration_fraction = 0.75
maximum__vegetation_density = 100
normalised__vegetation_density = vegetation_density/maximum__vegetation_density
reduction_in__regeneration = GRAPH(normalised__vegetation_density)
(0.00, 1.00), (0.1, 1.00), (0.2, 0.987), (0.3, 0.943), (0.4, 0.877), (0.5, 0.789), (0.6, 0.661), (0.7, 0.507), (0.8, 0.344), (0.9, 0.185), (1.00, 0.00)
vegetation_density = Vegetation/Area

- *What is the maximum sustainable limit of the vegetation growth in your model (in biomass)? Does it agree with the numbers above?*

  The vegetation rises to 100,000 biomass. That is 1000 hectares worth of vegetation at maximum density of 100 biomass per hectare, 100 X 1000.



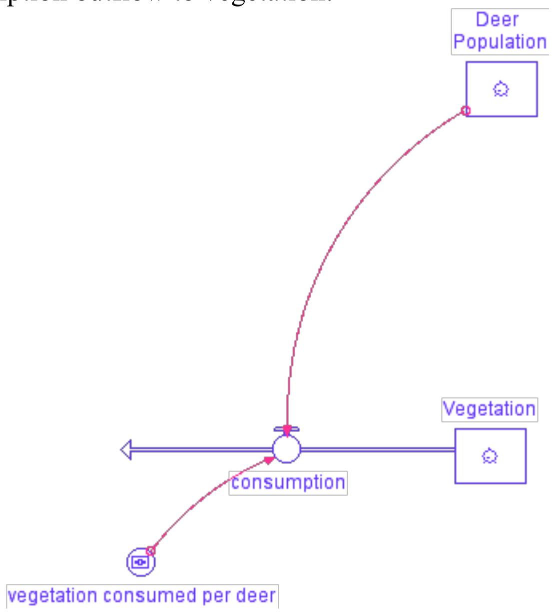
- *<u>Identify</u> the loops in the above model and relate them to the graph of vegetation.*

  The direct loop from vegetation to regeneration is reinforcing and is responsible for the accelerating growth of the vegetation. The loop via vegetation density is balancing. It is responsible for the slow down in growth.

  The model of the deer growth and the model of vegetation growth are both examples of limits to growth, though the reasons for the limit are slightly different.

Exercise 4

a) Ensure you add a consumption outflow to vegetation:

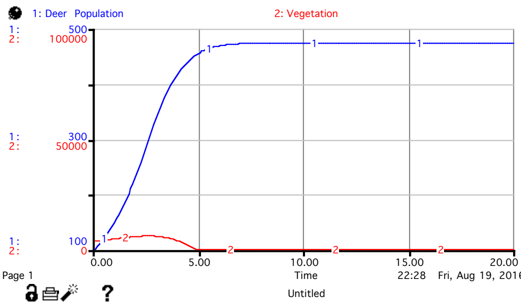


consumption = Deer__Population*vegetation_consumed_per_deer
vegetation_consumed_per_deer = 15

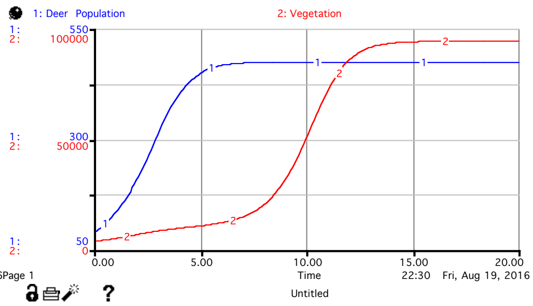
- *Are there starting levels of the deer population that cause the vegetation to survive?*

  Yes. 100 deer initially is too much for the vegetation to sustain growth. Any starting value of 90 or less should be sufficient for the vegetation to survive.

Initial deer = 100



Initial deer = 90



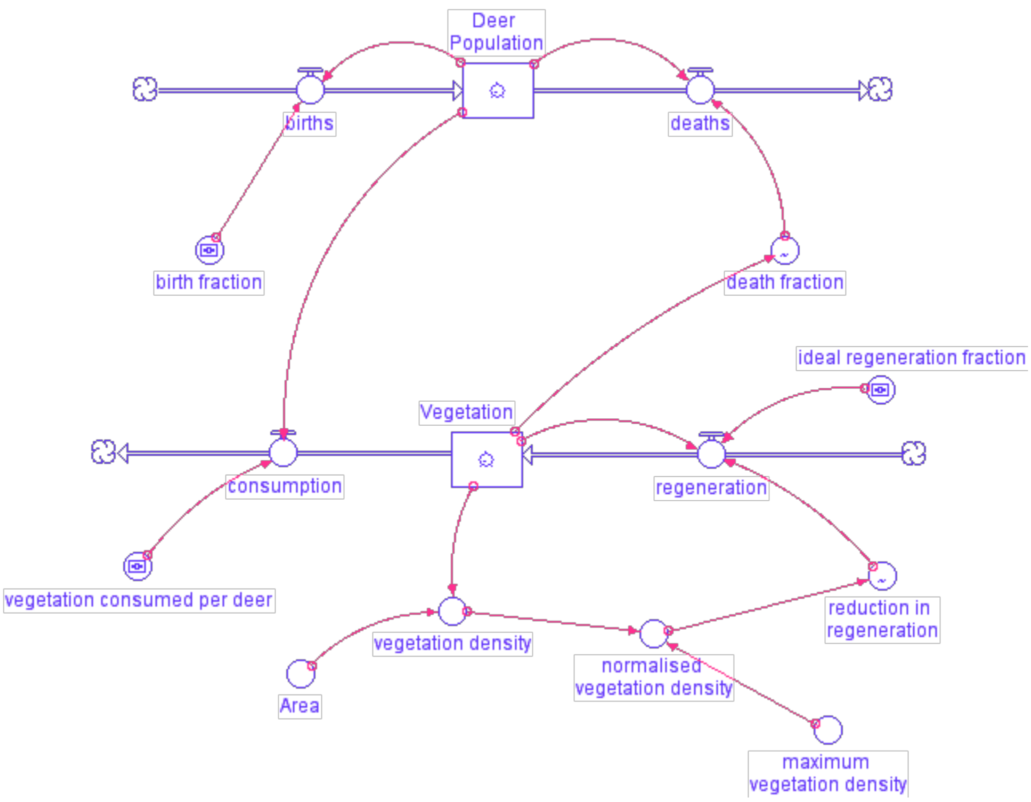
- *Are there starting levels of the deer population that cause the vegetation to survive?*

  Yes. Any starting value of 3780 hectares of vegetation is enough for it to survive with 100 initial deer.

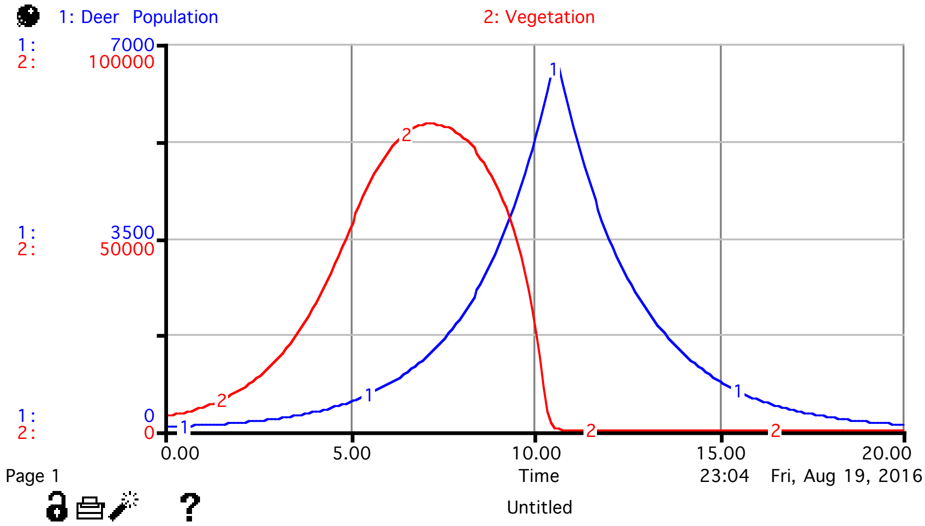
  Both the above experiments show that consumption of vegetation by deer risks the collapse of the vegetation. That is there is a critical point where vegetation cannot survive, for the given parameters.

b) Equations:
Deer__Population(t) = Deer__Population(t - dt) + (births - deaths) * dt
INIT Deer__Population = 50
INFLOWS:
births = birth_fraction*Deer__Population
OUTFLOWS:
deaths = Deer__Population*death_fraction
Vegetation(t) = Vegetation(t - dt) + (regeneration - consumption) * dt
INIT Vegetation = 3500
INFLOWS:
regeneration = ideal_regeneration_fraction*Vegetation*reduction_in__regeneration
OUTFLOWS:
consumption = Deer__Population*vegetation_consumed_per_deer
Area = 1000
birth_fraction = 1.6/2*(2/3)
death_fraction = GRAPH(Vegetation)
(0.00, 1.00), (100, 0.737), (200, 0.562), (300, 0.456), (400, 0.401), (500, 0.346), (600, 0.29), (700, 0.23), (800, 0.175), (900, 0.115), (1000, 0.067)
ideal_regeneration_fraction = 0.75
maximum__vegetation_density = 100
normalised__vegetation_density = vegetation_density/maximum__vegetation_density
reduction_in__regeneration = GRAPH(normalised__vegetation_density)
(0.00, 1.00), (0.1, 1.00), (0.2, 0.987), (0.3, 0.943), (0.4, 0.877), (0.5, 0.789), (0.6, 0.661), (0.7, 0.507), (0.8, 0.344), (0.9, 0.185), (1.00, 0.00)
vegetation_consumed_per_deer = 15
vegetation_density = Vegetation/Area

You may of course have slightly different values for the death fraction, as long as it starts at (0.00, 1.00) and ends at (1000, 0.067) and is monotonically decreasing. You may have more than 11 points.



With the above graphic converters the graphs are:



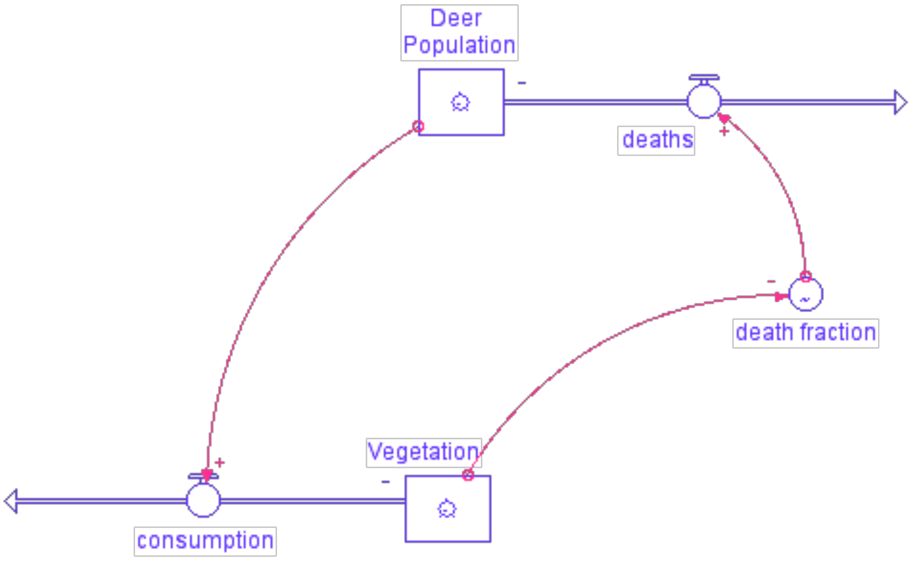
- *How many years before the vegetation reaches its peak?* *About 7 years.*
- *How many more years before it is wiped out?* *About 10.5 years*
- *What sort of advanced warning was there of impending doom?*

There is no warning in the behaviour of the deer, other than its growth keeps accelerating. However the vegetation is slowing too fast by 5.5 years for it to reach its carrying capacity. The inability of the resource to reach capacity could bee seen as advanced warning, but there is very little.

- *What was the vegetation's maximum value?* . *Almost 80,000 biomass*
- *When did the deer population start going downs?* *Around 10.5 years*

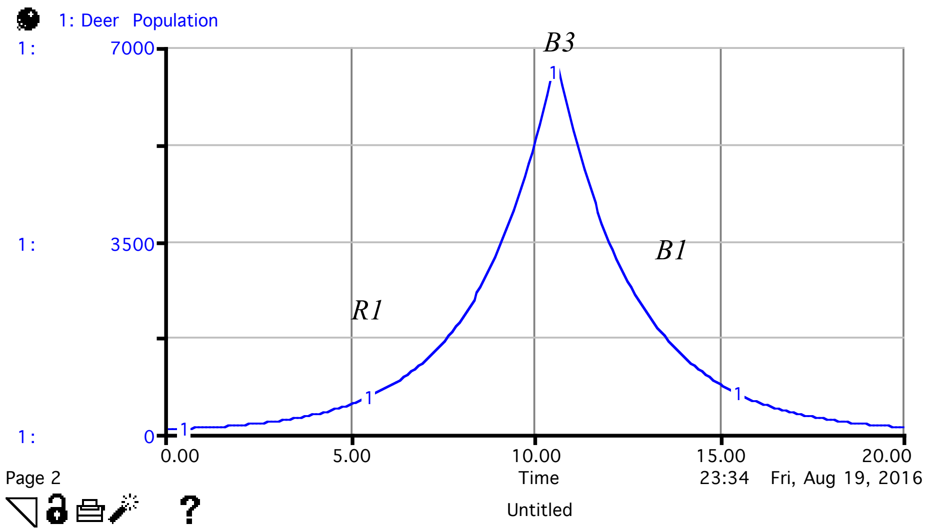
Feedback

- *By identifying link polarities* *<u>show that</u>* *the second order loop in the deer vegetation model is balancing.*



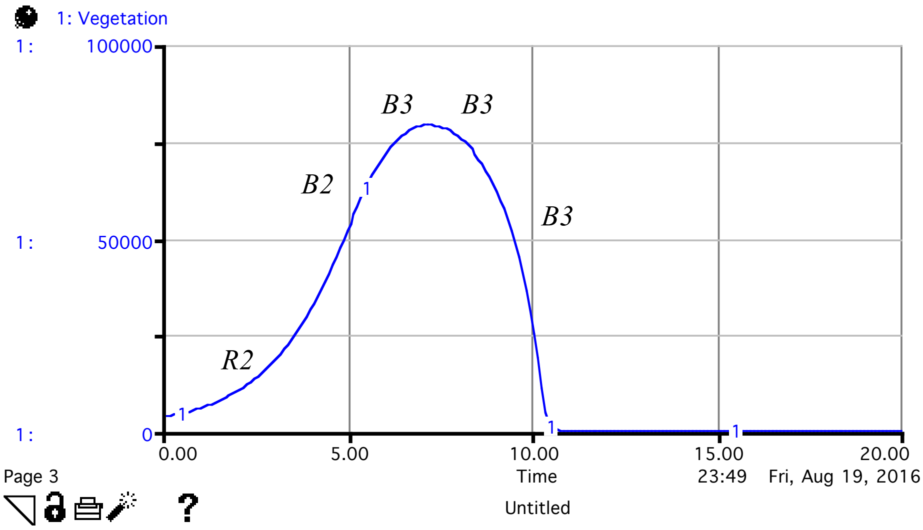
There are 3 negative links thus balancing. A better way is to say increasing deer, increases consumption, thus reduces vegetation, thus increases death fraction (opposite way link), thus increases deaths, and thus reduces the deer. Strictly speaking the last link says subtracts more deer, thus reduces its *growth* to a *lower value* than it would have been.

- *<u>Identify</u> the effects of the loops on the behaviour of Deer and Vegetation.*



The dominance of *B3* on the deer is very short lived. A possible failing of the model. The effect of *B3* is the effect of the vegetation on the deer. It is influencing the deer behaviour by reducing the level of its exponential growth, but *B3* only dominates while it turns deer growth into decline.

By contrast the dominance of *B3* on the vegetation lasts longer. That is the deer population is having a greater effect on the vegetation than the other way around.



How is a balancing loop causing the vegetation to *accelerate* downwards? You might be tempted to think balancing means slowing down, not speeding up. That is true for first order loops, but not second order.

Think of a balancing loop as negative polarity. A second order loops has two effects, one on each stock. The product of the polarities of these effects must be the polarity of the loop. For a balancing loop it means its effect on one stock is positive and the other is negative (plus times minus = minus). Thus there are periods when the effect of a second order balancing loop on a stock will be positive, i.e. accelerating. If it dominates, then it will cause the stock to accelerate.

Essentially second order balancing feedback has a delay built in, thus the attempt to balance responds too late to achieve equilibrium, but instead will oscillate about it. In this model the oscillation is interrupted by extinction!

Avoiding Catastrophe

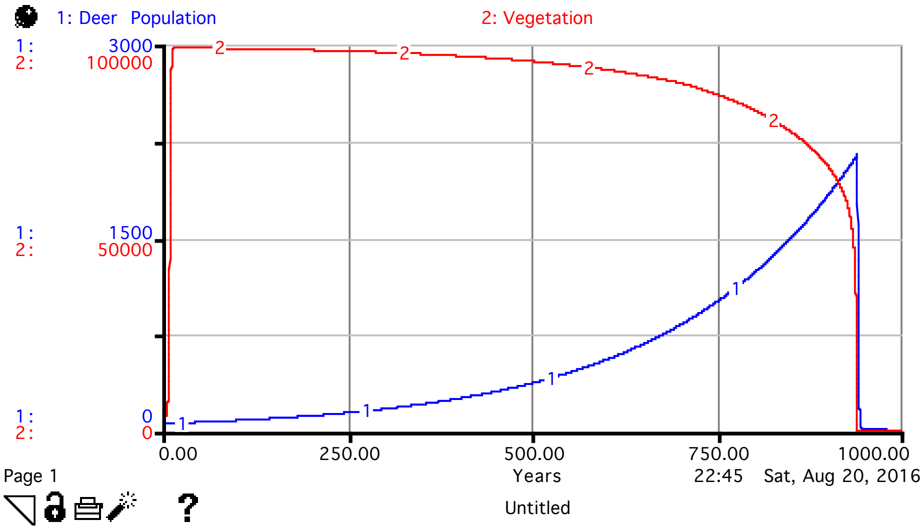
- *Which parameters do you think you have some control over?*

It could be argued there is some control over the birth rate through sterilising deer, or culling young deer. Neither straightforward.

If additional food were provided for the deer, then they would not need to consume as much vegetation per deer, so this parameter could be changed.

- *If so <u>change</u> the values to see if you can get the deer and the vegetation to survive.*

The lowest value of birth fraction that allows the deer to grow is 0.07. The deer just collapse later:



If the birth rate is any lower they become extinct straight away. Of course the timescale above means the deer numbers are reasonable stable for a human lifetime, but the massive reduction in reproduction is unrealistic.

Likewise reducing food consumed per deer merely delays the collapse.

Overshoot and collapse is only solved with a structural change in the model.

- *Try one or more of the above three solutions with constant flows - can you establish survival?*

It is unlikely you will achieve this. Constant flows take no account of numbers, thus there is no feedback and thus no control.

- *Try one of the above solutions with such a variable flow. Remember if you link from a rate (flows and some converters) you must smooth because they are instantaneous (see Stella Guide 4). Can you achieve survival?*

It is possible you will achieve control if culling is linked to deer numbers and vegetation numbers. The difficulty is how those numbers could be accurately obtained. A combination of culling and reintroducing or protecting vegetation by fencing the deer may prove more successful.