Mathematical Modeling of Church Growth

Explanatory Notes

Version 1.1, September 14, 2000

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Abstract

In these notes I attempt to provide an explanation of the paper Mathematical Modeling of Church Growth. Chapter 0 gives some background to the writing of the paper, my background, mathematical modelling, and revival in the Christian church. In subsequent chapters I follow the chapter divisions of the paper and provide a commentary on those parts of the paper that need further explanation. These comments may be of particular help to those with no understanding of the mathematics of the paper, but are interested in its relationship to church growth and the work of God in his church.

Older Versions:

Version 1.0, February 18, 2000

Contents

0   Introduction To Notes

0.1   Purpose of The Explanatory Notes

My paper "Mathematical Modeling of Church Growth"[1] attempts to use mathematics to bring understanding on the way churches grow and decline in numbers. As churches contain people there would appear to be no reason why the sort of mathematical models used in population modelling could not be applied, with appropriate modifications, to growing churches. The paper has been published in the Journal of Mathematical Sociology[2], although part of it was originally distributed in a technical report of the University of Glamorgan[3]. As such the work is now part of the academic communities of mathematics and sociology.

The purpose of writing these explanatory notes is to counter the feelings of unease that such a paper may produce within the Christian Church. The paper is academic, and uses mathematics, which is generally a foreign language to most people, including Christians. Thus those who may be the most interested in its results and claims will find large parts of the paper unintelligible. The paper also encompasses sociology, a subject often distrusted by believing Christians as many sociologists are antagonistic to religion, especially anything supernatural. Further still the paper leans on ideas from the church growth community whilst at the same time discussing the hallowed subject of revival. Many Christians who are passionate about revival deeply distrust church growth methodology. They fear that it has compromised the supernatural and Biblical foundations of the church replacing it with pragmatic and secular principles. Thus the paper runs the risk of being seriously misunderstood by those I was seeking to help the most.

It is also quite possible that mathematicians and sociologists may have problems with this work, especially with some of the more overtly Christian references. However as academics are used to controversy in research I suspect that they can take it in their stride and need no such reassurances from me. I have no doubt that some will pull the work to pieces!

As such these explanatory notes are directed at Christian believers, those who have faith in God, who is in control of all things - including growing churches. I hope these notes will show that mathematical models of church growth do not undermine our belief in such a sovereign and personal God, nor in his essential work in the conversion of people to Christ.

0.2   Reasons for Writing The Paper

My reasons for considering this sort of work, and hence writing the paper "Mathematical Modeling of Church Growth" are rooted in some of the claims made in the Christian Church, claims which on closer inspection cannot be substantiated. For example it is sometimes claimed that a "successful "church is one that sees many conversions, and if revival comes, the number of conversions are anticipated to be huge. The argument then sometimes runs that

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[1]   Hereafter called *the paper*.

[2]   Hayward (1999), Mathematical Modeling of Church Growth, Journal of Mathematical Sociology, Vol. 23(4), pp.255-292.

[3]   Hayward (1995), Mathematical Modelling of Church Growth, Internal Technical Report UG-M-95-3, Division of Mathematics, University of Glamorgan.

if the conversions dry up then the church has ceased to be successful, or its revival has stopped.

Without any resort to mathematics two serious <u>flaws</u> can be seen in these claims:

1   If most people in a community have been converted then the number of converts is bound to decrease as there are not enough people left to convert! The reduction in conversions cannot be an indication of God's disfavour or even of him removing a special blessing.

2   If the revived church is a very small part of a large unbelieving community then a large number of conversions would seem unlikely early on as believers physically cannot get the message to everyone fast enough. *How shall they believe in him of whom they have not heard?*[4] Given that the church started with the 120 at Pentecost, then vast numbers of conversions all over the Roman Empire could not happen within 6 months, even if the Lord intended to save every single person the gospel was spoken to straight away. People simply do not have that number of contacts that fast.

Effects like these are familiar enough in population modelling and successfully tackled with suitable models. Because the above effects are dynamical in nature, i.e. connected with the way population numbers change, it means that such numbers cannot be used as a predictor of what God is doing. The absence of a large number of conversions does not mean that God is not working positively in his church.

As a mathematician familiar with these effects in population modelling I realised that the models could give useful insights into some church growth situations if they were ever needed. Although I investigated some of the models privately from about 1987-1994, there never seemed to be any reasons to bring the results to a wider Christian audience.

However in 1994 Christianity hit the secular headlines with the "Toronto Blessing" affecting large parts of Canada, USA and the UK. This was heralded by some as a revival and by others as a deception, with various shades of view in between. One of the objections to the Toronto Blessing being a revival was that there were not a large number of converts[5]. It was this claim that encouraged me to write the original technical report[6]. I knew from population modelling that it was too early in the event to expect a large number of conversions. (See flaw 2 above.) It was not my purpose to defend the Toronto Blessing but I could see the same arguments could be used against any past revival, and would no doubt be used against any future ones.

Thus in the technical report I wished to make it clear that changes in the numbers within the Christian church follow understandable patterns even when they are due to a saving work of God. As such the lack of a large number of conversions early on is no indicator that some work is not a work of God, or that it will not eventually result in a large number of conversions. God usually converts people through other people and this affects the patterns of growth.

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4   Romans 10:14 NKJV

5   There are other objections to the Toronto Blessing being a work of God, all of which are countered by its supporters. Discussion of the genuineness of the Toronto Blessing, or whether it is a true revival, is strictly outside the scope of the paper and these explanatory notes. There are numerous books on the subject.

6   Hayward (1995).

My technical report was then turned into a paper intended for publication in the mathematical press. The reason for desiring external publication was that I wanted to make sure that the mathematical arguments were sound. Although some Christian mathematicians had looked over the work it needed the opinion of those actively involved in research in similar fields. This is provided by the anonymous referee system of a research journal. The journal of Mathematical Sociology has a history of applying mathematics in human situations, and has an international reputation for high quality work. Thus my work was scrutinised at a high level.

This scrutiny forced changes, largely due to my ignorance of the sociology of religion and the inroads it has made in church growth. Although not a large field, it was enough to change the feel of the paper and the reference list. However I was keen to ensure that my work is not just about a human situation but human responses to a divine intervention. I hope that the paper does not deny that emphasis, even if it is no longer so obvious.

0.3   Author's Background

I will briefly describe my educational and personal backgrounds so that the reader of the paper can see what sort of person should choose to mix Christianity and mathematics.

My education is strictly scientific. My first degree was in astrophysics and my doctorate in mathematics working on gravitation and general relativity. Having spent some years researching in solar physics, I eventually settled to being a mathematics lecturer, first at Napier College in Edinburgh (1982-9) and then at the University of Glamorgan (1989-present). It was while lecturing a science course at Napier College that I developed an interest in population modelling[7].

As for my personal upbringing I attended church as a child, but left it at the age of fourteen as a convinced atheist. However my belief system was challenged in the late 70's by people who accepted the Bible as true and the ultimate source of authority. Being a scientist I tried to counter this by proving the Bible was wrong, but in 1980 it proved me wrong and I became a Christian. I have never had any doubt that this was a work of God and not of my own doing or those around me. It also gave me a very high regard for the authority of scripture.

Being a Christian has not stopped me being a scientist and this inevitably affects the way I think. However at no time would I ever regard science as the authority, merely a tool to understanding. The only source of reliable authority is scripture - it never changes. Science is often wrong as history testifies. This is why scientific theories have to be changed so much! Nevertheless science is not to be feared. The more I understand how the world works through science the more I see God's hand at work.

I have found that these mathematical insights into the growth of the church help understand certain situations that arise, but it is still God giving the growth[8]. All I have done is give some insights into *how* he does it, *given* that he uses people. Once the insights cease to be helpful or they undermine scripture they can be dropped. They are not set in "tablets of stone".

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[7]   I am indebted to a former colleague, Mr GR Henderson, whose enthusiasm for mathematical modelling sparked my interest in this subject.

[8]   Matthew 16:8, Acts 3:47, and 1 Corinthians 3:6 leave us in no doubt *who* grows the church.

0.4   Purpose of Mathematical Modelling

To help understand the limits of the church growth model it might help if the reader can understand what mathematical modelling is, and why it is used. This will take us into the realm of physics, where mathematics has its most successful applications. This section may require a bit of persistence.

0.4.1 Definition of Mathematical Modelling

A model of any real-world object or situation is an attempt to describe that object or situation by certain key features of interest, whilst discarding those features which are not of interest. A mathematical model is such a model described using mathematics, usually with the purpose of explaining why something behaves the way it does, discovering some laws or patterns, and maybe making predictions. Thus a model has a purpose and mathematics is merely the language which enables the understanding and purpose to be expressed quantitatively and precisely. Purpose is essential for modelling.

This process of discarding uninteresting features is called abstraction and assumes one has a context for what is interesting or what is not! Context is also essential for model construction. In the case of mathematics, anything that cannot be expressed quantitatively is jettisoned since it is outside the bounds of the model.

An example of a non-mathematical model would be a scale model of a car. Its purpose is to achieve a miniature replica. In doing so many features of the real-world original are abandoned such as a working engine!

Much of science involves modelling. Every model inevitably has some compromise built in, mathematical models are no exception. What is compromised depends on the situation being modelled

0.4.2 Fundamental Physics

The best known mathematical models are those of fundamental physics. Typical of these are Newton's laws of motion. They can be expressed mathematically and are perhaps the most extensively tested of all laws of science. So much so no mechanical engineer would ever doubt them and, together with Newton's law of gravitation, were used to send Apollo 11 to the moon and back without the slightest doubt that the model would break down. To call such theories "models" is almost underselling something which seems part of the way God made the universe. In fact they are usually referred to as "theories" - models with a high degree of confidence.

In fact Newton's theories break down in three areas: high speed - solved by special relativity; small distances - solved by quantum theory and special relativity; and high acceleration - solved by general relativity. There is as yet no theory that covers all scenarios, however a search for such a model (sometimes called a unified field theory) continues with the assumption that one must be found[9].

However not all modelling concerns such fundamental laws of the universe. Even given such laws, further modelling needs to be done in order to handle solvability and complexity.

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[9]   Indeed this confidence is believed with an almost religious certainty, often by people with no belief in God whatsoever. That the universe is explainable would come as no surprise to Christians who believe that it was made by a rational being, but why do atheists believe it?

0.4.3 Modelling to Handle Solvability

Given the fundamental laws of the universe simple situations can be described and predicted with very high accuracy, for example the motion of one planet orbiting the sun. However, extend this to *many* planets orbiting the sun then, although it can be *described* precisely with mathematics, it cannot be *solved*, the equations are too hard. Thus further modelling needs to take place in order to achieve any predictability out of the model. This further modelling is achieved by making assumptions[10]. Thus there is now a less than perfect model of the real world situation in order that some answers can be found. It started with a fundamental law but assumptions are needed to simplify the mathematics and obtain a result.

Such modelling is familiar in engineering. Although based on Newton's laws, complicated arrangements of machinery are governed by equations that need further assumptions to be made in order to solve them. The arrangement of simple things can be made so complicated that mathematics (which is essentially simple) does not have the power to produce perfect results. However with time and effort the results can usually be made accurate enough.

0.4.4 Modelling to Handle Complexity

This complexity can be taken many stages further when living things are examined. Consider population modelling. Populations are composed of people, who are made up of basic chemicals, which in turn are composed of particles that obey fundamental laws. However the situation of a growing population is so far removed from the underlying fundamental model that any attempt to *derive* behaviour from this has to be abandoned as the situation is so complex. Instead a model is started from scratch based on observation and some sweeping assumptions. These assumptions are to reduce complexity.

For example the simplest model of a growing population is the "exponential law", which can be expressed as: "a population will double in number in a fixed interval of time". The USA population 1790 - 1850 doubled in number about every 15 years. This "law" is easily derived from the assumption that, on average, family size is constant.

Most mathematical modelling comes down to this type of "empirical" modelling, whether they are models of traffic flow, the economy or industrial processes[11]. Clearly such models cannot be thought of in the same light as the fundamental laws of the universe since they are so dependent on the assumptions made. For example the assumption of constant family size is dependent on people's behaviour (very unpredictable) as well as outside factors such famine, disease etc. Thus the exponential law will only ever have approximate validity for short periods.

If such empirical models have such a limited validity why are they constructed? The main reason is that they increase understanding of the situation being modelled, either numerically, or in the form of a principle. Thus in the case of the exponential law, it becomes clear that if family size remains fixed, and the population is growing, then it will ultimately get ridiculously large. The model will give some indication of the time scale on which it occurs. If it takes thousands of years it is no problem, if it is a hundred years it will need dealing with. Thus these models can make limited numerical predictions.

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[10]   In the case of the many-body problem a possible assumption is that the planets are all very small compared to the sun. Clearly this will not always be true.

[11]   Empirical modelling can also be handled by a method called Systems Dynamics, a diagrammatic way of constructing models which are then analysed using feedback and simulation. See Hayward 2000a.

However there is also a principle. The only way to stop a population growing exponentially is to reduce family size! Even if no numbers can be predicted some strategy can now be tried that effectively reduces family size. Perhaps different strategies can be built into the model and results compared.

The church growth model is this type of empirical model. It makes no claims to being a fundamental law of the universe. Neither does it make any attempt to predict human behaviour, let alone God's influence on the growth of his church. All it says is that given certain assumptions, then a certain type of growth results. These assumptions nearly always reduce down to saying "given God continues to act in the same way", or "people continue to act in the same way". No attempt is made to model how those ways might change.

Empirical models have for many years had a certain amount of success in modelling complex social situations, provided one realises they are not fundamental laws and that the models are very limited in their area of application. The advantage of such models is, as stated in the paper, the ability to make predictions, discover principles and examine the effects of changing parameters and strategies. The disadvantage is that people think the models are saying something more profound than they really are. The mathematics is merely an attempt to describe, in quantitative terms, the processes we see. It does not explain how the process is caused.

0.5   Objections To Church Growth Analysis

Church Growth, as a subject, has flourished in recent years with many books and journal articles. For a selection of such books British readers are directed to the British Church Growth Association[12] who produce a booklist, and publish a journal. The subject contains a mixture of theology, data gathering and analysis[13], sociology and pragmatism. For some Christians there is too much pragmatism and the whole subject is rejected. Indeed strong arguments are made against any form of church growth analysis, even data gathering. This has been defended by a number of people, for example Gibbs (1985) p106-111. These are theological problems outside the scope of this report. The reader should consult Gibbs and other relevant books.

0.6   Understanding Revival

0.6.1 Definitions of Revival

Some thought needs to be given to the relationship between the church growth model in the paper and the phenomena of revival. This is a complex issue as there are many definitions of revival. Some definitions are theological in nature centering on what God does, whilst some definitions give more weight to the effects produced in revival on people or the community. Consider the following definitions:

1   An outpouring of the Holy Spirit (many authors including Orr (2000) p.5) with reference to Acts 2:17.

2   A repetition of Pentecost (Lloyd-Jones [32, "the Way To Revival", p278] [33, ch.8, p100], Orr (2000) p.5).

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[12]   British Church Growth Association, http://www.personal.u-net.com/~the-park/NewFiles/bcga.html

[13]   I am not aware of any other author that has used mathematics, other than the statistical analysis of data.

3   The Holy Spirit comes down upon a number of people together (Lloyd-Jones [33, ch.8, p100]).

4   A community saturated with God (Duncan Campbell).[14]

5   A remarkable increase in the spiritual life of God's people, accompanied by an awesome awareness of the presence of God, intensity of prayer and praise, a deep conviction of sin with a passionate longing for holiness and unusual effectiveness in evangelism, leading to the salvation of many unbelievers (Edwards [14, p28]).

6   A manifestation of God's glory (many authors).

7   A sovereign work of God in which God pours out his Spirit on all flesh (Riss 1997 p.1)

8   Extraordinary work of a sovereign God that revitalises the church and transforms society (Greig 1998 p.xv).

9   An outpouring of the Holy Spirit, brought about by the intercession of Christ, resulting in a new degree of life in the churches and a widespread movement of grace among the un-converted (Murray 1998 p23-4).

Clearly some of these statements do not mean the same thing. A community saturated with God could adequately describe some situations in Old Testament Israel when God made his glory known[15]. However these could not be described as a "repetition of Pentecost". Pentecost hadn't taken place! They are not even an "outpouring of the Spirit", as the Spirit was only given, in the sense of his anointing, to selected individuals before Pentecost. However all definitions are applicable to the situation in the Christian church. All definitions can be summed up by saying that God has done something to people.

0.6.2 Revival and Church Growth

The popular view of revival is that it adds many people to the church. Strictly speaking this is a result of revival not revival itself. Riss (1994) has strongly emphasised this in the wake of the Toronto blessing[16]. Revival is something that happens in the lives of people, both believers and unbelievers, although in the latter case it is often referred to as an awakening. If such revival happens in a large number of unbelievers, growth of the church results. However if revival is largely confined to believers then the growth, initially, will be in the numbers of "revived" Christians within the church. This has been the story of charismatic renewal of the 1960's to the present, as well as the recent Toronto blessing. Both movements have led to substantial growth in the number of "charismatic" influenced Christians, who have come from within the church, rather than converts from outside.

In fact churches can grow numerically without any divine intervention at all! It is possible for churches to attract "social converts" - those who have adopted Christianity as their religion but for whom there is no supernatural or spiritual change within. It shouldn't happen, but it does, and it is often difficult to tell the difference between real converts, who have been

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[14]   Quoted in Edwards [14, p.26]

[15]   e.g. 2 Chronicles 5:14;7:1-3.

[16]   Indeed his comments to me in a personal communication made me change from using the "threshold of re-vival" to the "threshold of *revival-type growth*" in the paper in order to avoid any confusion.

changed by God, and social ones[17]. In times of revival this effect can get worse, particularly if churches fail to test converts. Some Christians struggle to recognise the existence of social conversions, however without them there is no other way of explaining the growth of other religions. I doubt if any of us who are Christians would attribute the rapid growth of Mormons, Jehovah's Witnesses or Islam to an outpouring of the Holy Spirit! The dynamics of growth of these groups is the same as the Christian church, but the source of the growth is different.

0.6.3 The Church Growth Model and Revival

As far as I can see the model of church growth in the paper would describe "revival-type" growth whether it was spiritual conversion, or social conversion. The key feature is that belief is spread through contact. Since God normally changes people through the preaching of the Gospel, and via the witness of Christians, then "spread through contact" is true, even for spiritual conversion. In the case where God changes people directly without the normal means then the model would not apply.

A change in the growth patterns comes through a change in the parameters of the model[18]. The parameters of the model depend on the effective contact between the believer and the unbeliever. This is an averaging effect over a large number of people but must ultimately be determined by how much actual contact takes place, the nature of the contact (what is said by, or seen in, the believer), and how receptive the unbeliever is to change. In the case of social conversion, the latter would be some psychological change in the unbeliever. The other changes in the believer would have to be interpreted in terms of their enthusiasm, but it would be difficult to find a model for the cause of their changed behaviour, particularly if such changes occurred suddenly.

In the case of spiritual conversion, which is the main interest of the paper, the changes in the believer and the unbeliever come about by God's activity. Note that the more involved definitions of revival (especially definition 9 above) pick up on this dual change, both in the believer, which makes them an enthusiast, and the unbeliever which brings about their salvation. This view depends on a theological model of conversion which sees the cause of a person's conversion lie with God, but where the means of their conversion is the interaction between a believer who witnesses and the unbeliever whose heart is opened by God. The mathematical model then merely describes the growth that takes place given a combined average value for these effects, but cannot say how that average value is achieved, or whether it stays constant over time or location. As stated before the model only works given that God continues to act in the same way. The cause of the changes in believers and unbelievers lies firmly with God.

0.6.4 The Church Growth Model and Conversion

This leads to a significant controversy in connection with revival governing the extent of God's involvement and human involvement in conversion. In the early years of the 19th century in the USA a different theological model for conversion arose which placed a greater emphasis on a person's ability to change their mind regarding Christian things and become

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[17]   The presence of social converts, those without a true spiritual change, is the context of the parable of the weeds Matt 13:24-30, and the parable of the sower Matt 13:1-23.

[18]   That is the parameters γ (gamma) and β (beta).

spiritually changed by an act of their will. This controversy, often associated with Charles Finney and called the "new measures" controversy, led to a change in the conduct of Christian meetings where more human "pressure" was exerted to secure a conversion. Such meetings came to be called revival meetings and has led to the word "revival" being used in the USA to mean any type of evangelistic meeting[19]. The UK churches have not generally adopted this use of the word revival (even if they agree with the new measures theology) and only use revival if some significant spiritual change takes place. My paper uses "revival" in the older use of the word, although the model for growth could be used for any non-revival situation that involves evangelism.

The new measures beliefs still hold considerable sway in the Christian church, although most people would probably place themselves on a point some where between the old and the new views. I have no doubt proponents of new measures will be able to interpret the church growth model in terms of their beliefs, but I think they will have a harder time interpreting the make up of the parameters, just as they have a harder time distinguishing social converts from spiritual ones. In the paper I have assumed that the changes in both believers and unbelievers are brought about by God even if he uses other people as instruments. Following Murray (1998) I think this fits the Biblical evidence better.

0.6.5 The Church Growth Model & the Reviving Work of the Spirit

A further controversy concerns the change that takes place in the believer. Is it a fixed effect moving from "ordinary" Christian to a "Spirit-filled" one[20], or is it something that occurs in varying degrees? (Undoubtedly the enthusiasm and ability in witnessing can occur in different degrees, even in the same person. In the model this is reflected in β (beta) being able to take a variety values.) Murray (1998) argues that the underlying work of the Spirit which causes these effects also occurs in different degrees. Even putting aside any Biblical arguments this view makes sense. It would be hard to see how a fixed spiritual change could make different degrees of changes in the same person, unless the person themselves has an effect on those changes. Revival, in Murray's view, is not taking the church back to where it should be, but giving more than a normal influence of the Holy Sprit leading to an increase in spiritual fervour, which is ultimately used by God in more conversions. Whether a "normal" Christian can ever be interpreted in any absolute sense or is only an average of recent experience is something I have never seen adequately tackled. In the model of church growth there is no definition of normal values for the parameters. Rapid growth comes from positive changes in those parameters which lies in positive changes in believers and unbelievers. Thus no definition of a "normal" Christian is needed to interpret the model.

The model does use two categories, S and I. These can be treated as averages of those with a "normal" work of the Holy Spirit and those with the "exceptional" work. The use of two categories is purely to simplify the model and ultimately the mathematics and in no way implies two classes of Christians. The only attribute of these categories modelled is their effective witness, how many conversions the people are used in. There may well be people in S

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[19] Readers are referred to Murray (1994) for a discussion of old measures versus new measures and the effect on the understanding of revival.

[20] Some argue that the "Spirit-filled" Christian is the "normal" one, and the others are "sub-standard". As these judgements are often made by those who regard themselves as Spirit-filled it can lead to an overdose of pride! This model makes no such value-judgement on the categories of believers. The adjective "Sprit-filled" does not occur in scripture.

for which this differs little from some in I. But on average there is a difference, the average being taken as representative of the whole.

As with spiritual changes in a believer, and their effective witness, there is also a sliding scale of revival, and there is bound to be a grey area where it is difficult to classify an event as a revival. It is worth remembering that "revival" is our word to explain what we see. At all times God is working in different degrees. If he executes such large changes that people are left in no doubt that something different is happening we are more likely to call it a revival. In the paper I only talk about revival-type growth, which is strictly defined by an increase in the net rate of infection - this is the threshold of the epidemic. I suspect there are many situations where this occurs that we would not call a revival. The model only models changes in numbers, not the spiritual experience of Christians.

0.7   Organisation of The Notes

From this point on the chapter headings follow the paper, as published in the Journal of Mathematical Sociology. Thus under 1.1 "par 2" refers to the second paragraph of that section. The page number (e.g. p.256) in the journal is also given as an additional reference. The notes take the form of a commentary, thus names, equations, phrases, concepts etc. are highlighted and appropriate comments follow. References by name and date alone refer to those given at the end of these notes. References by number refer to the original paper.

1   Introduction

1.1   Background to Church Growth

Donald McGavran (par. 1, p.255) McGavran is often referred to as the "Father of the Church Growth Movement". He was a missionary himself. His ideas were applied to the mission field before they were applied to home churches.

Two Strands of Church Growth (par. 2, p.256) The social science strand is mainly published in journals and conference proceedings. By contrast the Church Growth Movement publishes mainly through popular Christian books which are less technical. The British Church Growth Association (formed in 1981) publishes its own quarterly journal which is more in-depth than many of the books. The Christian Research Association (UK) conducts research in many areas of church growth in order to foster good practice within churches.

Sociology and Religion (par. 2, p.256) Reading the works of sociologists of religion such as Wallace [55] and Berger [6,7] quickly confirms Christianity's suspicion of sociology as they are very hostile to anything supernatural. Religion cannot be understood at the level of sociology alone, but on the grounds of scripture which is a higher authority than anything produced by man. Spiritual things can only be understood by a spiritual mind[21], thus secular theories of religion are bound to be found wanting.

Fuller Theological Seminary (par. 3, p.256) Church growth was part of the School of World Mission at this seminary. Other people who have served at the school and done much to popularise Church Growth concepts include Peter Wagner, Eddie Gibbs and the late John Wimber. The latter made use of the principles when building up the Vineyard churches of which he became the leader. The school also became known for its course on "The Miraculous and Church Growth", taught by Wimber, an approach which has influenced churches the world over and is popularly known as power evangelism.

Kelly (par. 4, p.256) Reference 24. The paper is a small book and is quite readable even for those not familiar with church growth concepts. His thesis that conservative churches grow and liberal ones decline would be accepted within evangelical churches, but some of his reasons for this thesis would differ.

Institutional Factors (par. 5, p.257) These would include the quality of the church programme, preaching, Sunday school work, evangelistic work, pastoral support etc., i.e. things under the control of the church leadership. They may also include things like quality of friendship networks, the fullness of a church service, things over which the leadership has no direct control. These are the areas the Church Growth Movement concentrate on as they are areas which can be improved in order to remove barriers to growth. Thus, in this scenario, someone might say that the church is growing, because it has Biblical and relevant teaching and that the people are friendly, or perhaps the worship is accessible. The social science strand has tended to ignore these areas until recently. These references include some institutional input [20] [27] [42] [50].

Contextual Factors (par. 5, p.257) This emphasises factors in surrounding society, such as poverty, wealth, class, race, etc. as reasons for growth. Thus, in this scenario, someone might say that Third World churches are growing because they have neither the wealth or education

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21   1 Corinthians 2:10-16, 2 Corinthians 4:3-6

of the more "mature" West, where churches are declining - the secularisation theory. Or they may say a congregation is growing because it is in a respectable middle-class area where people seek the moral stability provided by the church. This is familiar territory to sociologists. Perhaps they have failed to give due recognition to Institutional factors because they have failed to see that organisations such as churches are <u>subsystems</u> of people, whose institutional factors are contextual <u>within that subsystem</u>, and that some factors are under varying degrees of control of individuals or subsystems of people.

Secularisation Theory (par. 6, p.257) Essentially when people become educated and wealthy they have no need of religion which was merely an invented support for primitive and poor people. Karl Marx typifies this approach describing religion as the "opium of the people". The current popularity of religion, especially Christianity, in the USA and Canada runs counter to this theory. Whether the religious economy and rational choice approach is an improvement is debatable - it appears to have no spiritual justification. But at least some sociologists are now recognising that people need something beyond the material regardless of education and wealth, something which Christian believers have known for two thousand years!

Revival as facts of history (par. 6, p.257) Few would doubt that revivals of religion are facts of history but they are usually dismissed as mass psychology and not worthy of serious study. It is not clear why that view does not make it worth studying. The real reason maybe that sociologists have not regarded mechanisms underlying the growth of religion as important. It has been sufficient to note that its decline supports secularisation theory. However Ianaconne and Stark, as representative of the new approach to the sociology of religion, have regarded mechanisms of change as worthy of study [25-28] [50-53]. When this paper had been originally submitted I had missed their work entirely and I am grateful to the anonymous referees of the paper for pointing it out.

Statistics (par. 7, p.257) Although statistics is based on mathematics its use in modelling, and in church growth, is to analyse which variables may be related to each other and to give forecasts. It does not give understanding as to the causes of behaviour or underlying principles[22]. Much church growth data is so poor, (apart from the straight head-count), and depends on so many factors, its use is very limited.

1.2 Types of Models

Stochastic Model (par. 1, p.258) This is one which includes random components in order to model behaviour which is not predictable. It does not mean that the world is random and that one situation (or state) is not determined by a previous situation. It simply means that we cannot predict it with the information to hand, because there are so many factors. There are theoretical limits to predictability, due to the complexity of the real world, thus models with random components are often essential to simulate reality. The need for randomness says nothing about God's control over his creation, which scripture tells us is absolute. But it does say that created beings are limited in their ability to understand the creation of which they are part.

Deterministic Models (par. 1, p.258) Those where no random component has been included. They give average behaviour, but more importantly give the principles that help understand how the system behaves. I used such a model in the paper because my purpose was to increase understanding, not simulate reality.

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[22] Systems Dynamics is a good method for fleshing out the underlying causes of behaviour.

Epidemic Model (par. 2, p.258) Models used to describe how a disease such as chicken pox might spread through a population. They model the spread of the disease even when it is not of epidemic proportions. The model used here is highly simplified but contains the basic principles. They can be very sophisticated.

Oral Communication (Bullet 2, p.258) Communication by mass broadcast is not included in this model. It must be person to person.

Wales 1904 (Bullet 3, p.258) The 100,000 converts refers to those added to membership which is easily reconstructed from the data contained in church records, some of which are published in [58].

Have the Revival (Bullet 4, p.258) My own mother, who was born in 1908, knew people during the twenties and thirties who had experienced the revival of 1904. She could say of them that "you could see it in their eyes", and this was some twenty years after God had met with them in this way.

1.3   Diffusion in Populations

Non-Spatial (par. 1, p.259) There is no attempt to model how a church spreads geographically. This would be quite hard as it spreads along lines of communication, so it could reach a long distance quickly leaving nearer places untouched.

Logistic Model (par. 2, p.259) This is sometimes called S-shaped growth. Growth increases for a while but eventually slows down due to one or more factors that limit that growth. To be able to understand this sort of behaviour we need to go back to the simplest of models called exponential growth.

If there were no factors to limit growth the population would increase exponentially. For example the population may double every 40 years. As a graph this looks like:
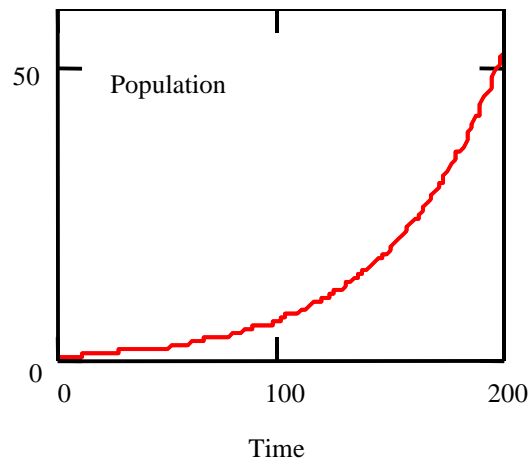
Exponential Growth

Not only is the population growing but the speed of its growth is increasing.

However if there is a limited food supply it will eventually slow down and will not exceed some maximum value called the carrying capacity:
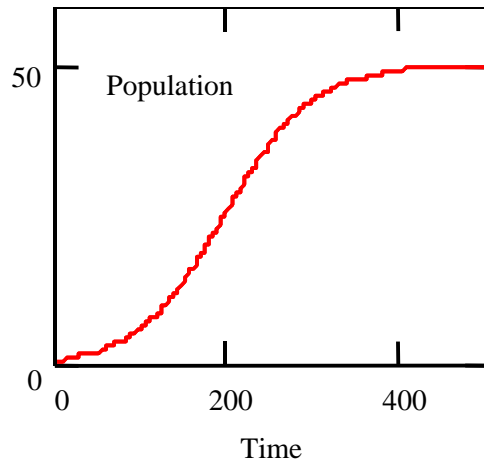
Logistic (S-shaped) Growth

This is the S-shaped growth. The adoption of innovations (e.g. colour TV) follows this pattern, the limit being the total population. Simple epidemics can follow this pattern leading to the whole population getting the disease.

In the general epidemic model of chapter 2 of the paper, and thus the church growth model, the "removed" follow this type of curve, but reaching a maximum some way short of the total population. The "susceptibles" decline in a similar way. The "infected" initially look as if they follow this curve but decline again due to their enthusiasm being limited in time:
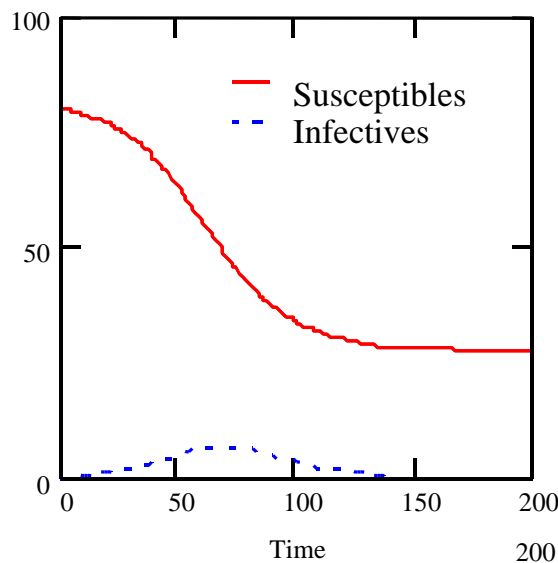
Epidemic

Bartholomew [5, Ch.8] (par. 3, p.260). I used the first edition of this book in the paper. However, unknown to me at the time, there was a later edition (Bartholomew, 1982) which has extended his work. Although mainly stochastic, there is much useful work in chapters 9 and 10 which I will no doubt use in further models.

1.4 Aims of Church Growth Modelling

Prediction (par. 1, p.260) There are too many unpredictable factors to use a model like this to determine the future numbers of the church. For example we cannot predict who God works

in, or whether people choose to change the way they evangelise. The model governs average effects over large numbers given that factors like these remain the same.

Fitting Data (par. 1, p.260) An attempt is made to take data from the 1904 Welsh revival and determine parameters, however this does not prove the model is correct. It says "if the model is true, what would typical values of the parameters be?" To prove the model from data, a record would be needed of the number of converts on say a weekly basis. Ideally the church or churches under consideration should represent a closed system, i.e. no-one comes in from the outside, or leaves. Reliable data of this sort is difficult to find.

Predator-Prey (bullet 1, p.260) A system of two species where one, the predator, lives by eating the other, the prey. This model was the earliest attempt at modelling interacting populations.

Principle (bullet 1, p.260) Establishing principles is one of the main aims of the church growth model. A principle is some qualitative property of a system which can be observed without having to put exact numbers to it. E.g. *if the birth rate exceeds the death rate the population grows exponentially*. The values do not matter. To give a principle that can be derived for church growth: *if a church has a small increase in the number converted per person, this leads to a huge increase in the numbers in the total church over time* - which is true under certain circumstances (near the threshold).

Dynamical Process (bullet 2, p.261) The process that describes how things change. The model helps understand how the numbers change. It may answer questions such as: why does a particular form of recruitment not lead to any substantial growth? The answers are not connected with numbers but with principles - maybe that particular form of recruitment leads to too much time being spent away from speaking the gospel to those who don't know it

What sort of data (bullet 3, p.261) This paper has not addressed the question of what sort of data should be collected. What is lacking is a monthly tally of who joins and leaves a church and why they do it. The last part of 4.1 in the paper describes the different processes of joining and leaving of which the church growth model describes only one. If the data could be collected by age group all the better, since different age groups interact in different ways.

Toronto Blessing (par. 3, p.261) In January 1994 thousands flocked to the Airport Vineyard Christian Fellowship[23] as people came under the power of the Holy Spirit accompanied by various phenomena. These phenomena were not new and were already present in the ministries of individuals in the USA, in the early Vineyard ministry of John Wimber, as well as among churches in Argentina. However the *scale* in Toronto was quite striking. It was this scale, rather than the unusualness of the phenomena, that initially attracted the attention of others. Special meetings continued at Toronto drawing people from around the world but not commanding any widespread attention. Indeed a number of people from the UK visited Toronto early in 1994 and were affected along with their churches, but few others in the UK knew about it. It was not till Eli Mumford of the South West London Vineyard church visited Toronto in May 1994, and similar phenomena occurred when she returned and spoke at Holy Trinity Brompton in London, did the phenomena become widely known in the UK[24].

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[23] The church left the Vineyard Movement towards the end of 1995 and dropped the word Vineyard from its title.

[24] For a history of the Toronto Blessing and its background see Riss (1995), and Dixon (1994).

The Toronto blessing has proved very controversial within the Christian church, with views varying from uncritical acceptance to claims that it is the work of the Devil[25]. It was extensively reported in the UK secular newspapers over the summer of 1994 who often seemed more sympathetic to it than the Christian church. The genuineness of the phenomena is beyond the scope of my paper or these notes. As such I have made no application of this model to the Toronto blessing other than the remarks about the number of conversions at the beginning of a revival. I have limited my applications to revivals of the past where the controversies are no longer remembered.

1.5 Overview of the Paper

From a church growth point of view the central claims of the paper are twofold:

1. The church grows through the work of enthusiasts within the church who alone are responsible for recruitment. These are the active or infected believers. Many Christians play no part in the church's numerical growth. These are the inactive believers.

2. The recruitment period of these enthusiasts is limited in time. This is not meant to imply that they lose something spiritual after that period, but that their recruitment potential ceases.

Both claims are open to objection. The paper assumes them and works out some of their consequences so that if they are rejected they can be rejected on clearer grounds.

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[25] All revivals within the Christian church have faced similar criticism, but the church has also had its fair share of deceptions as well. Discerning between them can sometimes be very difficult and is often easier to do long after they are over.

2 General Epidemic Model - Construction

2.1 General Assumptions

Anderson and May (par. 1, p.262) Anderson comes from a background of epidemiology, and has done considerable work on the spread of HIV. May has published widely on the mathematics of populations especially in the context of ecology. Their book [2] is the definitive treatment of the mathematics of the spread of disease and I have based the foundations of my model on theirs.

Latent Period (par. 4, p.262) Often there is a delay between a person acquiring a disease and being able to infect others, this is the latent period. More sophisticated models include it. Often the delay is so small compared to the infectious period that it can be ignored. I doubt if it is relevant in the church growth case.

Infectious Period (par. 4, p.262) This is the period a person is infectious for. However as people are often isolated when symptoms occur the effective period is often shorter. Of course people with colds and flu often do not isolate themselves and thus the disease is spread quicker. In the church growth case an enthusiast who is forced to be isolated is not of much interest. However I seem to recall it did happen to John Bunyan and the Apostle Paul!

Compartment Model (par. 4, Figure 1, p.263) In a compartment model the number leaving one compartment must be the same as that entering the next compartment. People are not "lost" from the system. Thus it is like a set of connected water tanks. Water can pass from one to the other, but none leaks outside or evaporates.

Transmission rate (par. 5, 263) How fast things move from one compartment to the next. How fast people are infected is the rate from S to I.

Homogeneous Mixing (par. 5, p.263) Infectives are equally spread throughout the susceptibles. If all the infectives are in one place (e.g. Glasgow) then people in other places (e.g. London) have no chance of being infected until someone changes location. In this situation geographical spread needs to be taken into account. This is typical of the spread of flu which affects some parts of the country before others. However small towns tend to become homogeneously mixed quite fast, and even the country can be regarded as mixed, on average, after a suitable length of time.

Equally likely to be infected (par. 5, p.263) Clearly people are not equally likely to be infected, this figure is an average chance of infection. Provided that the average for the whole population doesn't change over time, the average behaviour of the population is correctly modelled.

Equation 3 and last paragraph.(p.264) Ignoring the technical details, this is an example where a few simple assumptions can give a relationship, equation 3. Further assumptions now only concern one entity $n_i(t)$ the number of converts per infective. Thus further assumptions can be precisely stated. It is this separating out of assumptions that makes mathematics so good at forcing people to think clearly.

2.2 Crowd Model

A typical situation. (p. 264-5) One person with a cold enters a class of 30 students. Within the day sufficient contact is made with all in the class, to give some chance of passing the in-

fection on. If the class is 60 then twice as many people are likely to become infected. Of course if the class gets too big, 300 say, this assumption breaks down on the first day, because one person cannot make sensible contact with 300. But if after 3 days 10 people are infected, then the situation is now back to 1 in 30 (10 in 300). So doubling the class from 300 to 600 will double the number infected after a settling in period of a few days have passed. The crowd model nearly always comes into effect if you wait long enough and the infection takes off.

2.3   Fixed Contacts Model

A typical situation. (p.264-5) The situation of one student in 300 over the course of a day, mentioned above is typical, but it does not remain in this situation and quickly becomes the crowd model. However for sexually transmitted diseases, there are usually only a fixed number of contacts within a time period, depending on a person's behaviour. Much of the spread of sexually transmitted disease is driven by prostitutes whose number of different contacts will be more or less the same week by week because it is determined by available time. Thus doubling the susceptible population will not double the number of contacts with an individual.

The truth lies somewhere between the two models (par. 2, p.266) This is much harder to model. The answer to the question "how much between?" leads to at least one extra parameter. Too many parameters make models difficult to investigate.

3   General Epidemic Model - Results

3.1   Threshold of the Epidemic

Technically an epidemic occurs (par. 1, p.266) The spread of an infection is called an epidemic by mathematicians if the number new cases (infectives) each day goes up. In reality an infection's spread may not be classified as an epidemic unless the daily number of new infectives increases by a significant amount. This would be higher than the threshold. However the principle of the threshold still applies.

I_0 **does not influence the likelihood of an epidemic** (par. 2, p.267) This is a crucial result. It does not matter how few infectives there are, there will be an epidemic sooner or later. The likelihood of the epidemic is only determined by the number of contacts per person, the length of the infection, and the number of susceptibles.

3.2   Early Stages of an Epidemic

Exponential (par. 1, p.267) See the note on 1.3 under "Logistic model". Doubling the numbers each month is an example of exponential growth, as is compound interest. Once the growth gets going the rate rapidly increases. Many countries in the world still have exponential population growth.

Doubling Time (par. 1, p.268) The time taken for the population to double in number.

3.3   End of an Epidemic - Lack of Infectives

Equilibrium Point (par. 1, p.268) If the population variables attain these values then they will not move from them. The numbers will no longer change.

Stable (par. 1, p.268) If the equilibrium points are stable then even if the population variables haven't attained these values, they will get there eventually.

Epidemic ends (par. 3, p.267) A crucial result in epidemic theory. As people continue to get infected, more and more of the infective's contacts are with people already infected, or those removed (now immune), and less of their contacts are with susceptibles. Thus they increasingly waste their effort on people who will not become new infectives. The number of infectives drops and the situation continues to get worse. Some susceptibles are always left after the end of the epidemic.

4 Simple Church Growth Model

4.1 Use of the General Epidemic Model

Section (a)

Churches grow because of conversion. (p.270) Clearly they grow for other reasons, such as births and transfer, which are mentioned later in the paper. These are not included in the model in this paper, but are handled in Hayward 2000 a,b. .

Signs of Conversion. (p. 270) Church attendance, enthusiasm for the new faith, adoption of a moral code. These are changes that accompany conversion but in themselves are not proof that a genuine spiritual conversion has taken place. They could equally be signs of a purely social conversion, a work of man rather than of the Holy Spirit. It would take time for the differences to show. This model makes no attempt to distinguish real conversion from the temporal, it purely models the number of people who change category due to contact, whether there has been a work of the Holy Spirit or not.

Enthusiasm has a Significant Effect on Attendance. (p. 270) This could be for purely temporal reasons, people would rather mix with those who are enthusiastic. But it could also be for spiritual reasons: it pleases God to convert those who come into contact with believers who are full of joy and the Holy Spirit, rather than with believers whose lack of enthusiasm is a discredit to God.

Section (b)

Lead Someone to Christ. (p. 270) This term is not in scripture but is popularly used in many churches to describe a conversion where a believer has explained the gospel to an unbeliever and led them in a "prayer of commitment". As a method of evangelism this is a relatively recent innovation. In earlier times the contact would bring a person to a place where they could hear the gospel (one-to-one, or in a meeting), but the point of becoming a Christian would take place between the person and God alone, either in the meeting or at home, or even over a period of time, and not be linked with a particular prayer, decision or person. For example the Apostle Paul talked about preaching the gospel and about baptising people, but not about leading them to Christ or leading them to a commitment, a decision or in a prayer. See Murray (1994) for an extensive discussion on the issues raised. The expression was used in the paper purely because it would be readily understood by people within and outside the church, especially in the USA.

Active Believer. (p. 270) This is one who is actively involved in making contacts with unbelievers, for whatever reason, which leads to the conversion of some. They do not convert the unbeliever, but God does work through such people who give witness to the resurrection.[26] It is through these truths that God converts the unbeliever. It would be wrong to describe active believers as recruiters, as only God can convert. But if social conversion is taking place, as happens in a cult, the active people may well act like recruiters.

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[26] Acts 1:8. The primary message of Peter and Paul was the resurrection as a fact and a proof of all that the cross implies. Their motivation for preaching this message was based on people's standing with God in the light of these truths, rather than recruitment.

Section (c)

Not all the people are responsible for spreading the faith. (par. 1, p.270) This is meant in the active sense of specifically proclaiming truth. People may be involved in passively giving attention to the faith through their Christian lifestyle, which may make it easier for active believers to be instrumental in conversions. Thus a low percentage of active believers is not necessarily a criticism of the Church. God uses people in different ways. A more sophisticated model could take this into account.

Often it is new converts who are the most enthusiastic (par. 2, p.271) This is a modelling assumption. It is quite conceivable that enthusiasts could be made out of the removed category, i.e. inactive believers become active. This requires a more sophisticated model. Likewise some converts may never be an active believer, possibly due to shyness, or not being in a position to be open about their beliefs. However they have been deliberately excluded, so that the behaviour of this modelling assumption can be investigated uncluttered by other effects.

Wesley's Law (par. 2, p.271) This refers to John Wesley and is quoted from Kelly[24]. I do not know where in Wesley's writings it occurs.

Limited recruitment (par. 2 & 4, p.271) Recruitment is again used to mean spiritual conversion, or social conversion depending on the application of the model. If active believers could remain active indefinitely then, in this model, everyone in the population would get converted (see Hayward 2000a). In practice this would not be true either for spiritual conversion or social conversion, since there would be a residue of those hardened to any conversion. A sociologist would say they are resistant to the pressure. A Christian would say both that their heart was hardened <u>and</u> that God did not convert them.

Timescale (third par. from end, p.272) As long as the church is not large, 20 years would give reasonable results. However the important reason for ignoring births etc. is that it will be easier to understand the effect of limited recruitment. Part of the modelling method is to start with the simplest model and add in additional effects one by one. The principles determined will still stand even in more sophisticated models.

4.2 Identification of Variables and Parameters

Simple Epidemic Model (par. 1, p.272) The word "simple" is wrong. It should say "*general* epidemic model".

Susceptibles (S) (Bullet 1, p.272) They may include people who have some connection with church or none. They may have been born into a church background etc. It could be argued that different categories of believers are more susceptible than others. This could be true if some groups are in complete ignorance of the gospel and others already know the basics. However the idea that some people are more "convertible" than others is a sociological argument. It is my belief that if the truth of the gospel is explained to people in a way they can understand it and grasp what they should do about it, then all people are equally likely of conversion because it lies in the hands of God. Talking about the likelihood of a person being converted is just the recognition that we cannot predict what God will do - but there is nothing in the person that will either decrease or increase that likelihood. God can convert anyone.

Number of converts per infected (active) believer (par. 2, p.272-3) This is an average figure. Clearly it will vary among active believers. This averaging effect is taken into account in the way the equations are put together.

High tension with society (par. 2, p.273) This means that the church is very hostile to society, and society hostile to it. Thus contacts between them, of any sort, may be few, and the group is little trusted. This is one of the reasons why many cults such as the Moonies do not grow substantially. They find it difficult to recruit among people who have heard of them and distrust them because of their activities. A cult in this position may have to take steps to reduce that hostility and build respect if it wishes to grow. Both the Mormon church and the Jehovah's Witnesses have attempted to do this. The move is then from a cult to a sect and eventually to a church, where acceptance is widespread. However not all cults do this. It is possible for a cult to survive, albeit on small numbers.

Nothing To Offer (par. 2, p.273) The danger with moving too far in the direction of being acceptable to society is that the group loses most of its distinctive features which may well be some of its most worthwhile! For the Christian church the Gospel, particularly with its negative assessment of man without Christ in relation to God, is never going to be attractive to society. That is promised throughout scripture and is simply a result of sin which always seeks to exalt mankind. However the Christian church has frequently moved in the direction of "watering down" the message to make itself more attractive. A number of authors point out that the decline in Western Christianity is a result of the theological liberalism of the second half of the 19th century which continues to this day. It is often forgotten that liberalism was a well intentioned attempt to make the church more relevant to those outside. It may have improved the image, and even the number of contacts, but the number of *effective* contacts (conversions) declined (β smaller) as there was now little reason left for anyone joining the church.

Length of time a believer remains infected (par. 3, p.273) Again an average which is taken into account by the form of the equations.

4.3 Identification of the Transmission Mechanism

If the population of unbelievers is increased will the church grow faster? (par. 1, p.273) The various scenarios that could give rise to this follow in the paper. Note it is generally easier for a church to grow in a big city than in a small town, or village. An active believer is far more likely to come across new people in a big city, especially if the church is centrally based. Again if the church is in a fairly mobile community, the stock of people is continually replenished with new-comers. These types of communities are where some of the fastest growing churches can be found.

Social Networks (bullet 2, p.274) Despite the popularity of social networks in modern church growth strategies, there is very little evidence of them in the New Testament. The bulk of the evangelistic work in Acts was done amongst people unknown to the evangelists and we are usually left to surmise that news spread by word of mouth.

4.4 Interpretation of Epidemic Results

Revival Growth (par. 2, p.275) Revival growth means the sort of increase in numbers of believers often seen in revivals. In the context of the model it means the number of active believers increases. To be thought of as a revival in popular terms there would probably have to be a significant increase before it would be classed as one. The essence of a revival is the work of the Holy Spirit in individuals. It cannot be stressed enough that the equations <u>do not</u> model this. The model concerns the effect on the growth of the church <u>given</u> that work in be-

lievers and <u>given</u> it is passed on to new converts. The threshold is a threshold of growth given the spiritual conditions among people, not the threshold of a revival.

End of Growth (par. 4, p.275) Growth ends because enthusiasts spend an increasing proportion of their time with believers, as the number of unbelievers falls. Of course if enthusiasts deliberately keep seeking out unbelievers this will not happen. However most people who are caught up in a revival spend more and more time on activities generated by that revival and less time with unbelievers. Even evangelistically minded leaders may find an increasing proportion of their time is spent helping the new converts and that they have less time to spend with unbelievers.

10 enthusiasts responsible for 11 converts (par. 7, p.275-6) That is 1.1 converts per active believer. This may seem incredibly small, but it does give the indication that the average number of converts per infected believer during the whole of their enthusiastic period does not have to be high to give substantial growth.

To work out this average for an individual congregation, count the whole adult congregation[27] and assume all are active. Say it is 100. Count how many straight converts there have been in say two years (that is exclude those brought up in church families). Say it is 3, you may be surprised at how low it is! Thus $n_i(0) = 3 \div 100 = 0.03$ initially. Say that there are only 5 in the congregation who are active in the sense meant in this paper. $n_i(0)$ is still only $3 \div 5 = 0.6$. However this number has to be over 1 before there is any revival-type growth, because that is the only way the threshold can be less than the total population. The number of unbelievers has to be above the threshold for revival-type growth to occur.

Of course you can extend the length of the enthusiastic period in order to get a higher number of converts, claiming your own enthusiasts last longer than the typical. But now the church takes much longer to grow, with the effects being wiped out by the imbalance between births and deaths. The reality in most churches is that the number of converts is very low, too low for any long-term growth to be sustained.

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[27] Children and teenagers are excluded purely for convenience. Teenagers often come out of this exercise better than adults.

5 Numerical Solutions

5.1 Scaled Equations

Non-Linear. (par. 1, p.276) A "linear" relationship between two quantities means that if a percentage change is made in one quantity, the other quantity changes by the same percentage. Equations with this sort of relationship can be solved easily with the answer expressed as a formula. For equations where the relationship is more complicated, non-linear ones, such a solution may not be possible, as is the case with the epidemic model.

5.2 Increasing the Effectiveness of an Evangeliser

Explain the Gospel Effectively (par. 1, p.277) Because people have a better understanding of the things they believe they have more confidence to speak of these things to others, either making more contacts, or making better use of the contacts they have already made. Underlying this is the assumption that Christian witness often gives such a poor presentation of the truth that there is insufficient content for an unbeliever to make a sensible response.

The question has to be asked: if conversion is in the hands of God will improving the "effectiveness" of an evangeliser make any difference? To investigate this ask the question: do people who fail to make a response to the gospel, understand it enough to know the implications of making, or failing to make, a response? If the answer is yes, then none of these programs can improve the effectiveness of an evangeliser as measured by number of converts[28]. But if the answer is no, and I suspect the bulk of the population has little understanding of the Christian gospel, then ensuring that people can communicate the message with confidence will increase the <u>potential</u> for more converts. It is this potential the model investigates. If God still converts no-one, then the link cannot be made and the conclusion is not valid.

Seeker Church Model (par. 2, p.277) In this model the Sunday services of the church are given over to outreach and hence conducted in a way that unchurched people find comfortable and attractive. The church members are then able to invite their friends and relatives to a "service" which is understandable to non-Christians but still conveys the truth of the Gospel. The underlying assumption here is that people are unwilling to invite friends to a church service which is only intelligible to the initiated as its primary purpose is the worship of God. Thus, in the seeker model, people will invite more of their friends and thus make better use of their existing contacts. In addition there are many secular activities run by the church that enable members to establish new contacts with unbelievers.

These methods are not new. Many large churches in the 19th century would put on afternoon meetings which consisted of a non-threatening, and non-churchy, atmosphere suitable for those who didn't relate to conventional church. They usually met in a secular building. Additionally many churches ran secular activities in an attempt to make contacts in the community.

The current outreach method known as the Alpha course adopts a similar philosophy with regard to teaching Christian basics. People listen to a very low-key talk, or video, on some as-

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[28]   Only God can convert, and no amount of forcing the issue will make a true spiritual convert, only a social convert. However God does use the *means* of the Gospel. Is there enough of the Gospel there for him to use?

pect of being a Christian and can then debate it in a small group without any fear of criticism of their existing beliefs. People are free to drop out of the course without any pressure to return. Part of the Alpha approach is that the relationship building with unchurched people is as important as the teaching.

Cell Church (par. 2, p.277) In this philosophy the basic Christian unit is not the congregation, but the cell group (or housegroup) which can typically number from 6-12. This cell *is* the local church, and is the primary vehicle for evangelism. Thus members can no longer pass the responsibility for spreading the gospel onto others and so become more involved in exploiting their contacts for evangelism. The Sunday meeting is retained as the place for the combined worship of all the cells, but is no longer the main point of contact with unbelievers. New people are likely to be integrated into a group before "going to a church on Sunday." Interestingly, although I am sure Cell Churches will not like this analogy, the Jehovah's Witnesses (JW) use a similar strategy. New people start in JW Bible studies, and possibly an open meeting on a Sunday, before they can "graduate" to that church's main worship service.[29]

The Cell Church model, and the Seeker Church model, contrast quite sharply with the traditional model of the local church as a worshipping community defined by its Sunday worship. Both Cell and Seeker models are also defined by community. The Cell Church sees its community in terms of the relationships within the cell group, whereas the Seeker Church sees community within its program and networks of groups. In neither case is the Sunday meeting seen as the definition of the local church community.

Doubling the effectiveness of an individual more than doubles the growth rate of the church (par. 5, p.278) This works on the assumption that those converted have a similar effectiveness to those responsible for their conversion. Such new converts have an immediate impact on the growth of the church (all converts become I before they become R). In times of revival this nearly always seems to be the case. New converts are very infectious.

Of course this assumption may not be true. If new converts become "low-recruiters" then doubling effectiveness will only double the growth. It would be interesting to know how many people in the church are responsible for bringing people to a position where they can become Christians (any sort of contact from explaining the gospel to inviting people to a service.) My unscientific observation is that it is a very small minority of the church. This is precisely the point made by proponents of the Cell Church method.

We seem to suffer from two failures - we are not on fire enough and thus are not effective enough; and then we are not able to pass that enthusiasm onto the new-converts. New converts often get endued with the same timidity and lethargy that infects most of the church. If the fire could be passed on then starting with a small number of such believers would not matter. Large growth *would* come (the point of 5.4). Of course the fire needs to be passed to existing inactive believers as well, a feature missing from the current model.

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[29]   Both the Seeker model and the Cell Church model struggle to find significant examples of their strategies in the Acts of the Apostles where the main method of the spread of the gospel is by travelling evangelists and apostles who speak the basics of Christianity to strangers, possibly staying with those who respond long enough to see them established as a church. Interestingly, the Methodist fathers such as Wesley, Whitefield and Harris adopted the Biblical strategy. They did set up small groups - but for the edification of the converted. The conversions came through fairly orthodox preaching services either in churches or in the open-air. Presumably the existing believers invited their friends to these services, or, more likely, were responsible for spreading the news of such things which attracted the interest of unbelievers.

5.3 Increasing the Number of Evangelisers

Increasing the number of people involved in evangelism (par. 1, p.278) Both the Seeker model and the Cell Church model aim to increase the number of people involved in evangelism. This is the result of taking existing members from a very low effectiveness to a much higher effectiveness. The two methods (5.2, 5.3) are two sides of the same coin.

Increasing the number of evangelisers does not have the same impact as increasing an evangeliser's effectiveness. (par. 1, p.278)

Again this is only true if that effectiveness is passed onto new believers. However I cannot see how this can be done except by a work of the Holy Spirit. Spreading "the fire" is his work and we cannot produce this result by any training method, even those that improve a believer's effectiveness. Only if the Holy Spirit improves a believer's effectiveness through an outpouring or baptism of the Spirit upon them is there any way it could be transferred onto those converted through them. Of course this is what happens in revival, the Spirit falls on the new converts as well. Thus my conclusion from 5.3 and 5.2 should be more specific, and spiritual: a work of the Holy Spirit which makes believers more effective (on fire) will have a greater impact than any attempt by man to make *more people* with effective contacts, or to make people have *more contacts* which are effective. Even the latter cannot be guaranteed to pass their infectiousness on - only God can do this. The better growth of 5.2 over 5.3 is only valid if it is a work of the Spirit.

5.4 Medium Term Revival

Slow start behaviour (par. 5, p.280) The church in the UK is numerically a very similar proportion to society as the church of the 18th century. Thus the "slow start" behaviour is almost certainly to be the type of growth we could expect in a revival. Thus a revival could have started but its numerical effects may not yet be noticeable in the total numbers in the church. If the revival only effects a portion of the church, the increase in that portion may currently be swamped out by the decline of the old.

Figure 1 gives an example of this by comparing the new denominations[30] (which are largely charismatic and evangelical) with the traditional denominations[31] that date back two hundred years or more. Figures are available for England from 1975 up to 1989[32]. The new denominations are growing, but that growth has little effect on the total because of the decline of the traditional denominations where there is a much higher proportion of older people, and less evangelism. If these figures are projected forward then eventually the growth of the new churches dominates and the total church figures begin to rise again, about 2015. However the revival had started back in the 1970's (or even the 1960's), and it took over forty years for the total church to start growing.

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[30] Pentecostal churches and independent churches which includes the new charismatic and restoration streams, plus older independent evangelical groupings.

[31] Church of England (Anglican), Baptist, United Reformed and Methodist.
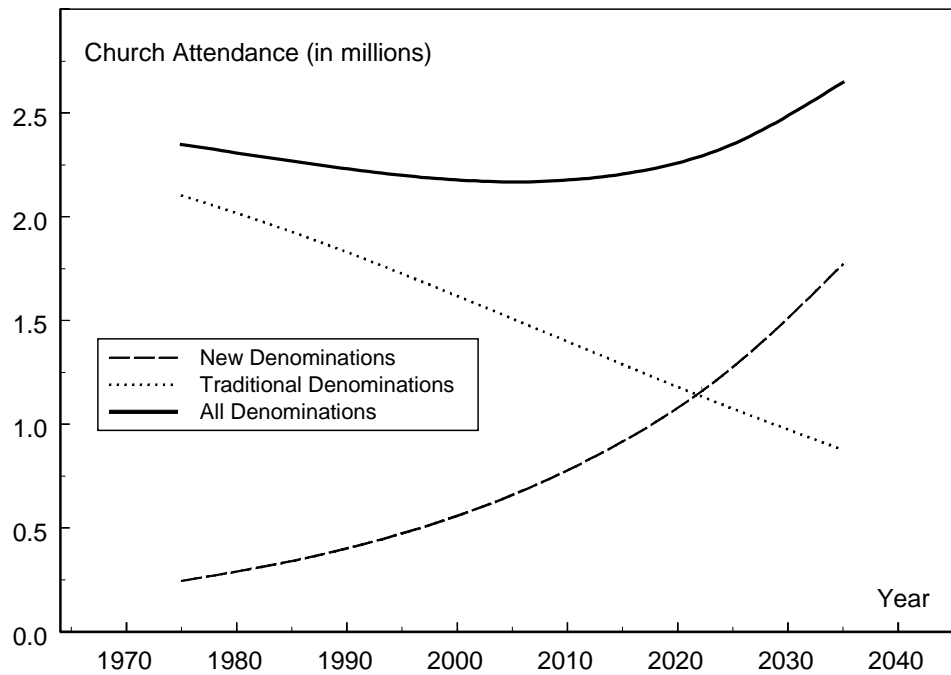
[32] Data is taken from Brierley (1991).

Figure 1 : Projections of Church Attendance In England
Traditional Denominations (Anglican, URC, Methodist, Baptist)
compared with New Denominations (Pentecostal, Non-Denominational, Independent)

WARNING. Do not quote figure 1 as a possible future! It is purely an illustration of how a revival in part of the church can take many years to affect figures for the whole. It carries little predictive power. Firstly, no errors for the extrapolation have been given - which would quickly swamp out any predictions. Secondly, it is unlikely that the new churches will continue to grow exponentially. Figures during the nineties from interim surveys suggest they are leveling out after the high growth of the eighties. Thirdly, there are considerable charismatic and evangelical wings in the Anglican and Baptist churches which are also growing, and may well lead to the growth of those denominations in time.

To repeat one of the conclusions of the paper: a movement within the church cannot be dismissed as a possibility for revival just because it fails to make the church grow. Underlying growth can be slow and take time to affect total figures. The genuineness of a revival must be determined on Biblical grounds in terms of the lives of the people affected.

Revival is ending due to dynamical effects (par. 6, p.281) The definition of revival given earlier in these notes is that God does some special work to some believers that makes them more effective, and that this results in converts who themselves receive this special work. Revival will cease if this special work ceases. In the model this appears as a reduction in the value of the parameter β. God may choose to do this because the church has in some way sinned, or quenched the Spirit. He may also withdraw it because he sees that the need for the special work is over - he does not choose to give revival in all circumstances, it doesn't have to end due to sin. An interesting question would be: "does he withdraw the blessing from all people at the same time in all places, or is it spread over time, people and places in a way we cannot easily predict?"

The point of dynamical effects is that if a believer's effectiveness is limited for a fixed time period, an average of all the different people involved, then the revival will eventually end without any parameters being changed. If God does withdraw revival blessing over time, peo-

ple and places in a way we cannot predict, then this is effectively contained in the model by the giving of an average value to $D = 1/\gamma$ the period of a believers enthusiastic phase. Thus the phrase "ending due to dynamical reasons" includes the result of God ending a revival due to his own sovereign will, or in response to human sin. The model only deals with the average effect, and not the individual details. Thus God is always in control of the progress of his church, the model merely describes the effects of his work.

It is not ending due to any change in spiritual conditions (par. 6, p.281) This is a bit misleading. The conditions that haven't changed are the extent to which an enthusiastic believer is effective, and how long they are effective for. Of course their numbers eventually decline away in this simplistic model, thus the spiritual climate of the church as a whole changes! I suspect that in a model which includes the effect of enthusiastic believers on other believers, and births and deaths, then a residual number of enthusiasts will always remain, and this would be larger after a revival had taken its course. This will await more sophisticated models[33].

Theologically a revival is regarded as an act of God (par. 7, p.282) By this I mean a revival <u>is</u> an act of God, a work of the Holy Spirit as amply indicated in the book of the Acts of the Apostles! I had to understate this fundamental truth because this work is published in an academic journal that embraces mathematics and sociology. Works of the Holy Sprit and the resurrection of Christ would be regarded in such circles as beliefs, not as facts, because scripture carries no authority in secular academic life. Personally I regard works of the Holy Spirit and the resurrection of Christ as undeniable facts, as proved by scripture whose authority is without question. They are more certain than any scientific theory - however good the theory - and I recommend them to any person reading these notes. Without a belief in these facts, and trust in the person of Jesus Christ, the results of my paper will ultimately be of no benefit to anyone at all.

5.5   Short term Revival

To summarise: if revival occurs in a community the larger the fraction of the community in the church initially, then the faster the revival takes place, the quicker it is noticed, and the sooner it is over.

5.6   Estimation of Parameters from Data

5.6.1 Details of the Figures for Church Membership in Wales 1904-5

The population for Wales is only known with any accuracy at 10 year intervals, following the UK census. In 1901 the adult population was 1,328,437, and in 1911 it was 1,623,463, an increase of 295,026. The population figures at intervening times were estimated by linear interpolation, i.e. the increase is equally divided between all 10 years 29,502.6 per year (see table 1, column 2). Although this is not strictly correct for exponentially growing populations, the long-term population of Wales does not fit an exponential. It was already slowing down and shortly after this period declines due to migration effects (see figure 2). More sophisticated attempts at fitting the population show little difference from linear interpolation for this period.

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[33]   The inclusion of births, deaths and reversion is included in the technical report Hayward 2000a.

Year	Adult Population Wales	Calvinist Methodist	Baptist	Wesleyan Methodist	Welsh Independents	Total	% of pop
1901	1328437	160333	109149	36288	129993	435763	32.80%
1902	1357940	162284	110955	36913	130252	440404	32.43%
1903	1387442	165218	113597	37679	131242	447736	32.27%
1904	1416945	173310	116310	38803	143341	471764	33.29%
1905	1446447	189164	140443	44016	152165	525788	36.35%
1906	1475950	187768	143584	44991	149750	526093	35.64%
1907	1505453	185935	137507	44122	147043	514607	34.18%
1908	1534955	185366	133287	43319	143819	505791	32.95%
1909	1564458	184558	130681	43246	142562	501047	32.03%
1910	1593960						
1911	1623463						

Table 1: Adult population of Wales 1901-1911, and known church membership
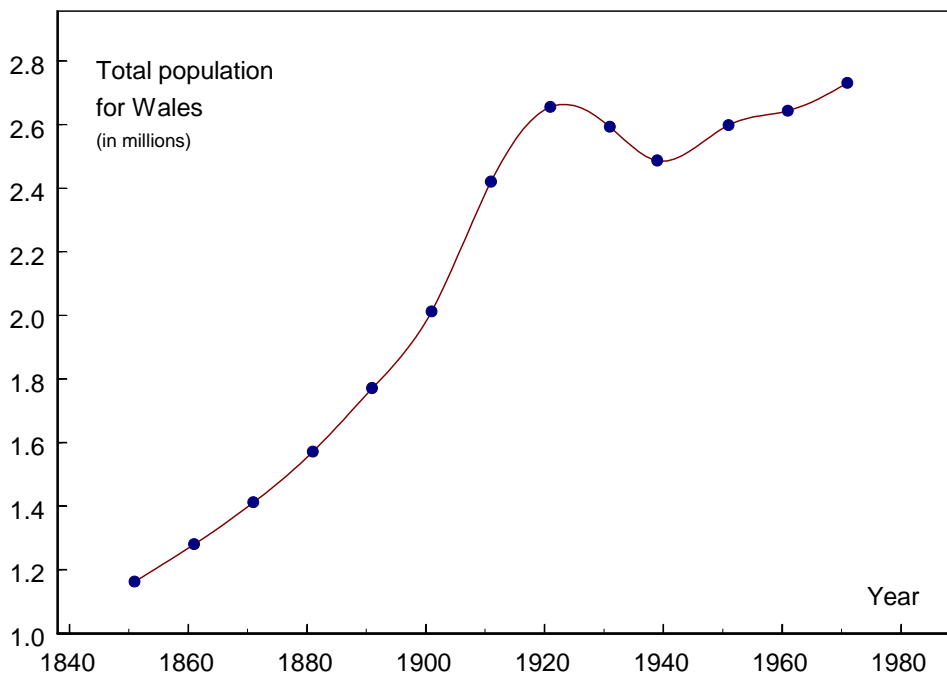

Figure 2: Total Population of Wales 1851-1971

The numbers of adults in the four main non-conformist denominations are known. Note the rapid increase over the period of the revival. It would appear by 1909 that all the numerical benefits of the revival were lost. This is not necessarily true, the growth of these denominations in percentage terms was already showing signs of decline (see figures 3 and 4), the re-

vival gave a momentary increase and delayed that decline until later. Without the revival they would probably have declined faster and sooner.
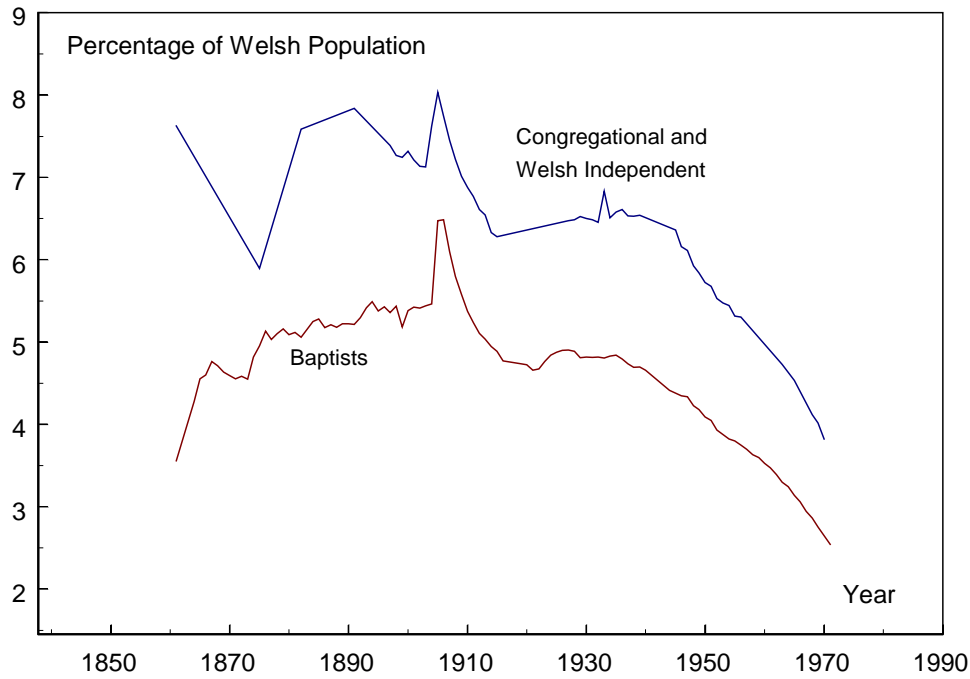

Figure3: Baptists and Welsh Independents as a Percentage of the Welsh Population
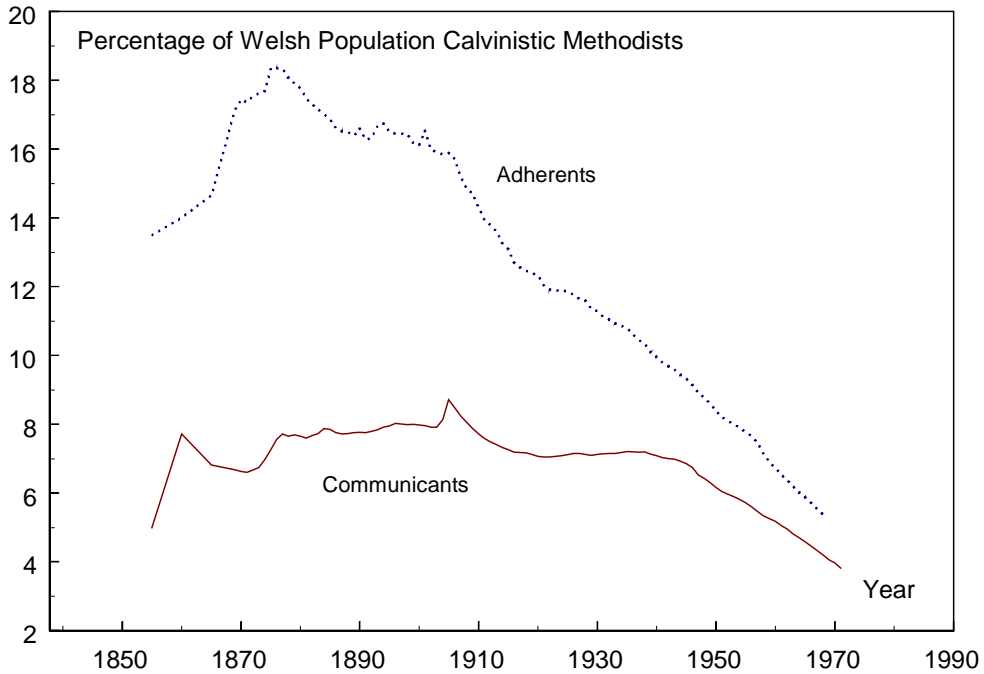

Figure4: Calvinist Methodists as a Percentage of the Welsh Population

In addition to these denominations there are three groupings whose figures are not known on a yearly basis at this time: Anglican church, other independent churches, and miscellaneous groups such as Brethren, Salvation Army etc. However a separate survey for 1905 alone

(Williams [58], Table: Religion 12) does give these numbers together with the denominations above (table 2).

Anglican	Miscellaneous	Calvinist Methodists	Baptist	Wesleyan Methodist	Welsh Independents	Other Independents
195004	21876	170348	142551	43358	175,095	22,930

Table 2: Communicant figures for 1905

Although these numbers do not square exactly with the denominations' own returns (they refer to communicants rather than members) it is assumed that they give an accurate *proportion* between the different groups. Thus the three missing groups can be estimated for the years 1904-1906. The one problem denomination are the Welsh Independents, for which this survey includes congregational churches not in the denominations' own figures. Thus the 175095 is an overestimate and the denomination's own figure. 152165 is taken instead, with the remaining 22930 treated as missing independents.

$$Known = 170348 + 142551 + 43358 + 152165 = 508422$$

$$Missing = 195004 + 22930 + 21876 = 239810$$

Thus the proportion of missing to known church numbers is 239810 ÷ 508422 about 0.47, allowing the numbers missing in each year to be estimated. Thus for 1905 there were 471764 in the known denominations and, therefore, $0.47 * 471764 = 221729$ in the missing ones. The total number of members of all Welsh churches can then be computed (table 3).

The reader should not be too worried about the estimated data. All that has happened is that the percentage increase in the four main non-conformist denominations has been applied to the rest. There is plenty of anecdotal evidence to support this claim. So even if the absolute numbers are not accurate the percentage change (last column, table 3) is unaffected and depends on known data.

Note that the effect on membership of the revival is well documented. Revivals are facts of history!

Year	Adult Population Wales	Total Known Churches	Estimate Missing Churches	Total Membership Welsh Churches	Percentage of Adult Population	Percentage Increase in Membership
1901	1328437	435763	204809	640572	48.22%	
1902	1357940	440404	206990	647394	47.67%	1.07%
1903	1387442	447736	210436	658172	47.44%	1.66%
1904	1416945	471764	221729	693493	48.94%	5.37%
1905	1446447	525788	247120	772908	53.43%	11.45%
1906	1475950	526093	247264	773357	52.40%	0.06%
1907	1505453	514607	241865	756472	50.25%	-2.18%
1908	1534955	505791	237722	743513	48.44%	-1.71%
1909	1564458	501047	235492	736539	47.08%	-0.94%

Table 3: Membership of all churches in Wales

5.6.2 Details of the Calculation of the Number of Converts Per Infective

1 in 500 of 693,493 is 1,386 which is near enough 1,500. Thus

$$i_0 = 1500 \div 1416945 \approx 0.1\% \, .$$

The basic church growth model is run with this value of i_0, $s_0 = 100\% - 48.9\% = 51.1\%$ in such a way that it is over in about 12 months with $s = 100\% - 53\% = 47\%$. A version of this simulation is given in appendix 1 using the Stella System Dynamics simulation software[34]. This determines the duration of the infection as about a week ($D = 0.25\text{months}$) and the number converted through one infected believer as just over 2 ($n_i(0) = 2.02$). Although the original computation was carried out with my own program, it is easier to present the model using the Stella software, which reproduces the same result[35].

$n_i(0)$ **would have to be at least 2** (last par, p.285) Equation 23 says that the number of susceptibles times the number of converts per infective must be at least 1. $s_0 \times n_i(0)$ must be at least 1. As half the population are susceptibles, s is a half, so the number of converts must be at least 2 to make a number bigger than 1.

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[34]   Stella is manufactured by High Performance Systems inc. and is distributed in the UK by Cognitus Ltd. Harrogate.

[35]   Stella uses a less accurate numerical method than my own software, but the effects on the result here are negligible.

6   Conclusion

6.1   Main Conclusions

Point 1. (p.286) See my notes on 5.2 and 5.3.

Point 2. (p. 286-7) See my notes on 5.4.

Point 4. (p. 287) The term "**wasted**" comes from diffusion theory and was suggested by one of the anonymous referees. It would mean more of the infected believer's activity was spent on those already converted. This does not imply this is a waste of time - it may be very beneficial. It purely means that it has no potential to convert unbelievers.

Point 5. Secularised. (p.285) Sociological models of church describe believers as becoming more secularised over time. Kelly, Stark and Ianaccone use this term. It means the believers become more like the world, and less religious. Secularised believers have virtually no involvement in conversions. This a long term process of the church rather than individuals, particularly noticeable comparing different generations. The children of believers who are often more secularised than their parents. However my claim is that believers lose their "infectiousness" long before secularisation sets in. In fact they may be more religious and more genuinely spiritual, but still have no impact on the world.

6.2   Further Work

Point 4. (p.288) Revivals nearly always start in the church so this process is essential to have a complete model. It raises the question: "Do enthusiastic believers have a bigger impact on inactive believers, or on unbelievers?" Indeed do we have some measure of choice in this balance? If, in a work of the Spirit, we enthuse of the things that effect us only to other Christians, then the dynamics described in this paper will not take place at all. Such a model could describe the effects of how this enthusiasm is channeled.

Point 5. (p.288) I would be very unhappy with a model that treated some people as more resistant than others. Conversion is a work of God and depends on his power, not our resistance. There was no-one more resistant than the Apostle Paul! Sociologists of religion do consider resistivity. However a group could be treated as being easier to reach because, for example, they were brought up in church, as compared with those who were not , or follow a completely different religion. Here it is matter of meaningful contact, rather than resistance in the person.

7   Acknowledgements

8   Further Clarification

If any person reading the paper or these notes requires further clarification I will be happy to respond to any questions or comments. Any omissions from these notes may be addressed in future revisions

Additional References

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# Appendix 1: Simulation of 1904 Welsh Revival

Stella Simulation of the Simple Church Growth Model Applied to The Welsh Revival of 1904-5

Duration of "infection" in an infected believer = 0.25 Months

Percentage of population in church = 48.9%

Percentage of the church infected = 0.2% (i0 = 0.1%)

Number converted through one infected believer = 2.02

Graph of Total Church (I and R) and Number of Non-Believers (S) - Wales 1904-5
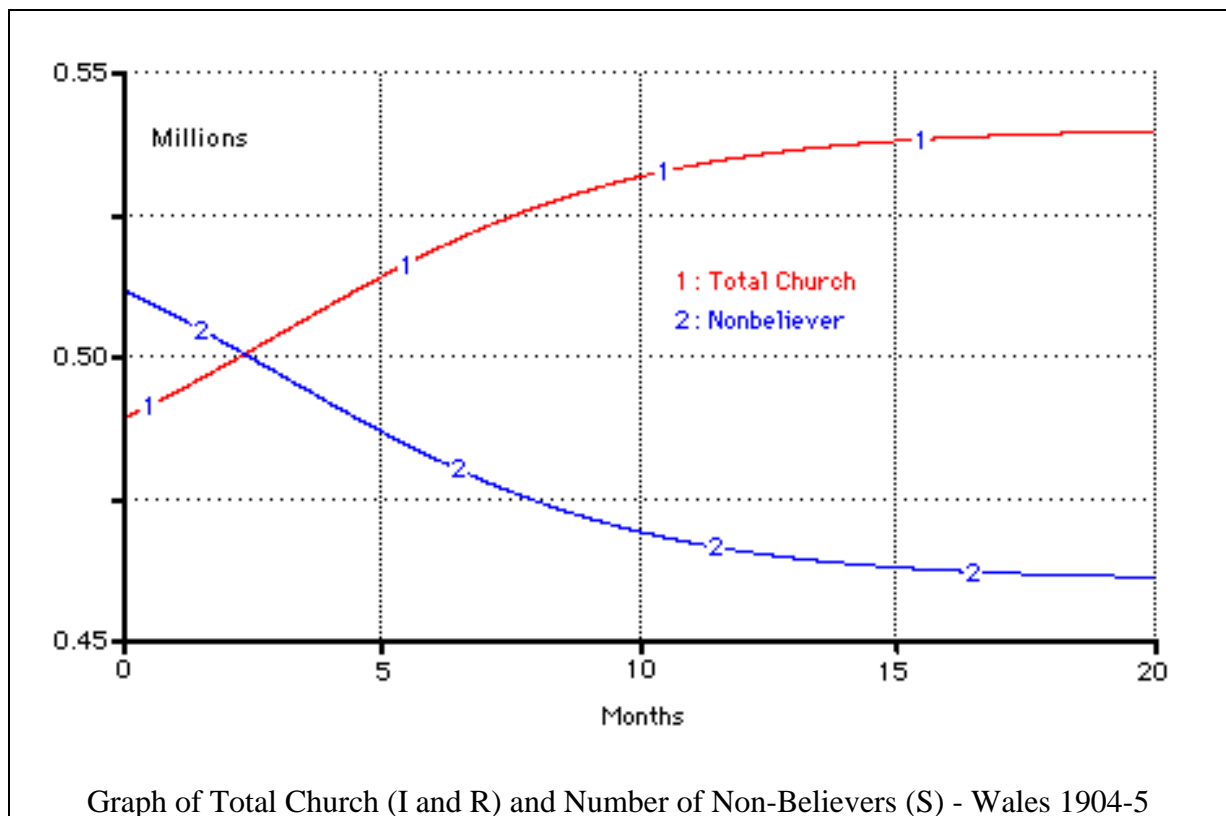
From the above graph the bulk of the growth was completed by the end of 1905, i.e. in 12 months. However a graph of the number of enthusiasts shows that they reached their peak after only 4 months when only a third of the growth had taken place. The bulk of the growth is occurring after the peak in the number of enthusiasts. This is typical of epidemic behaviour.

Graph of the Number of Infectives or Active Believers (Enthusiasts) - Wales 1904-5