Modelling Church Growth: A Systems Approach

Part 1

Introduction to Church growth, Systems Dynamics
& the Unlimited Enthusiasm Church Growth Model

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Abstract

The growth of a church is investigated using the notation of systems dynamics. It is hoped that this approach will be more accessible than a strict mathematical one.

Part 1: The systems dynamics method is explained from scratch (chapter 2). This is then applied to a simple model of church growth where the population is split between unbelievers and believers who are all enthusiasts involved in the conversion process. Conversion is driven by contact between enthusiasts and unbelievers (chapter 3). These enthusiasts are assumed to retain their conversion potential throughout their lifetime. As such the whole population gets converted to the church.

Part 2: The model is made more realistic by including births and deaths (chapter 4), and reversion (chapter 5). If the church fails to recruit all its own children, or if it loses people after conversion, then the whole population does not get converted. As such the church numbers settle at a value less than the total population depending on the strength of these effects. If the effects are extreme the church becomes extinct.

Part 3: The model is modified so that enthusiasts lose their enthusiasm, i.e. their conversion potential, after a length of time (chapter 6). The church is now split into enthusiasts and inactive believers. Again the whole of the population fails to be converted. Including birth death and reversion effects into this model compound these problems (chapter 7). The model is applied to the current state of the Christian Church and some past revivals.

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1 Introduction

In the last thirty years a considerable amount of effort has been expended in attempting to understand why and how churches grow or decline. Much of the work is qualitative in nature usually giving recommendations to a church as to how it may improve its growth. (Gibbs 1981, Pointer 1987, Schwartz 1998 are typical examples.) However quantitative measures of growth are also important. A church growth analysis of a congregation usually involves a survey of its current state and numbers are inevitably required. Some typical questions a church may wish to know are:

- how fast is my church growing?
- how many people leave?
- how many conversions are there?
- is my church becoming more spiritual?

Of course the latter question is hard to answer as there is no unique quantitative measure of what it means to be spiritual. However the other questions are much easier, because all that is involved is a count of people. As such this aspect of church growth is a branch of population modelling.

A church growth analysis is not limited to an individual congregation, but is also handled at the regional, denominational, and national levels (e.g. Brierley 1999). In some ways these become easier to handle than the congregational level because at these scales unpredictable effects tend to be smoothed out. Thus deterministic population modelling is adequate to describe the effects of different mechanisms for growth and elucidate the principles of a growing church.

Despite the similarities with population modelling most quantitative church growth work is either at the statistical analysis level (e.g. Hoge and Roozen, 1979) or numerical arguments (Stark 1996)[1]. However in Hayward (1995, 1999) a church growth model was developed using mathematical population modelling. The model, based on the general epidemic model, was constructed on two principles:

1. conversion growth of the church comes from contact between unbelievers and active believers, called enthusiasts;
2. enthusiasts only hold their enthusiasm, or potential for converting others, for a limited length of time after which they become inactive, although they are still believers and part of the church.

"Conversion" refers to the recruitment of people who have started off outside the church and, having believed, now become part of the church. The hypothesis that there is a category of believers called enthusiasts, splits the church into two sorts of believers: the enthusiasts, who are responsible for the recruitment; and the inactives, who have no involvement in recruitment.

The model exhibited growth behaviour very similar to that seen in a religious revival, where the growth runs out because there are insufficient enthusiasts to maintain the level of

[1] Hayward 1999 contains a longer discussion of the history of church growth research and its relationship to the sociology of religion. The church growth modelling website http://www.church-growth-modelling.org.uk also contains further information

conversion among a dwindling number of unbelievers. The result is that the growth ceases without all the unbelievers being converted. Indeed there is a threshold that the number of unbelievers must be above for significant growth to take place. That threshold depends only on the number of people that an enthusiast is responsible for converting during their enthusiastic phase (Hayward 1999 equation 23).

The disadvantage of the model is two-fold. Firstly, because the model is expressed in mathematics, the work is largely inaccessible to many of the people who would be interested in its results - namely those within the Christian church who are involved in church growth work.

Secondly a number of important processes are missing from the model, such as:

- births and deaths - biological growth;
- believers reverting back to unbelievers;
- effects of persecution.

In order to tackle the first disadvantage the model will be re-cast using system dynamics, which is diagrammatic method for understanding growth and decay. Although much of the detail of the model is hidden in this approach it has the advantage that the important structures that determine the type of growth are expressed in easy to follow diagrams. Thus the reasons for the growth behaviour can be understood in terms of the fundamental processes involved, without any need to see or understand the mathematical detail.

Once the method has been established (chapter 2) it will be applied to a simple model of conversion from the unbelieving society to the church. It will then be possible to look at the four effects of biological growth, reversion, persecution and limited enthusiasm on the growth of the church. The first three processes, which have a similar effect, are looked at in chapters 4 and 5. The effect of limited enthusiasm will be the same model as Hayward 1999 but now looked at from a system dynamics point of view. This will analysed in chapter 6, with all processes combined together for a full model in chapter 7.

2 Introduction to System Dynamics

2.1 Dynamic Model Building Blocks

There are a number of different approaches to system dynamics. The one presented here originated with J.W. Forrester under the name of Industrial Dynamics (Forrester 1961), and is the philosophy and notation behind the Stella software[2] used in this paper to simulate the models.

There are only four basic building blocks used in a system dynamics model: Stocks, flows, connectors and converters .

2.1.1 Stocks

Fundamental to a system are the Stocks, represented by:

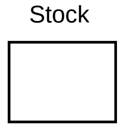

Fig. 2.1a. Symbol to represent a stock.

These represent the basic variables, or quantities that change in a system. For example in a population model one stock may represent the population of a country:

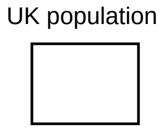

Fig. 2.1b. A Stock to represent the UK population.

A stock can be thought of like a water tank with its value given by the amount of water it contains. At one time UK population may have 55million, at a later time it may be 60 million. The stock would now contain more water:

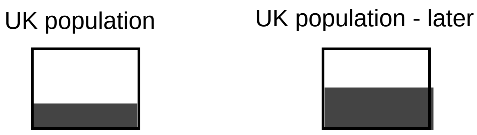

Fig. 2.1c. Two stocks using the "amount of water" to represent the population's value.

For a simple church growth model there would be one stock representing the number of unbelievers and another stock representing the number of believers:

[2] Stella is manufactured by High Performance Systems inc. and is distributed in the UK by Cognitus Ltd. Harrogate.

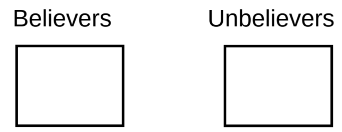

Fig. 2.1d. Stocks representing the number of believers and unbelievers.

These would be part of the same system at the same time.

2.1.2 Flows

Just as water can flow in and out of a tank, so quantities can flow in and out of a stock. For the UK population numbers can flow in through births and numbers flow out through death:

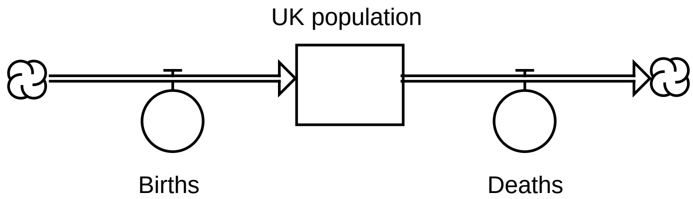

Fig. 2.1e. Flows representing mechanisms for changing the level of a stock.

The circles on the flows act like taps controlling how many go in and come out at any time. This is the rate of the flow. The clouds at the ends of the flows just indicate the outside world. In other words we are not concerned as to where the people come from or go to in this model.

For a church growth model consisting of two stocks, unbelievers and believers, a flow could connect one to another:

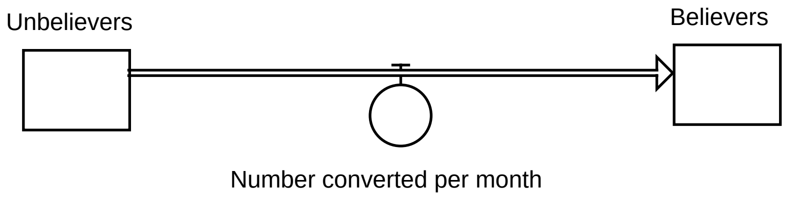

Fig. 2.1f. Unbelievers become believers through conversion.

The flow controls the number converted at any time. None are lost to the outside world in this model as people are either believers or unbelievers.

2.1.3 Connectors

A flow represents a physical link between stocks. However there are also information or dependency links. For the UK population the more the population the higher the number of births in any given year. Thus the rate of flow into the UK population depends on the population:

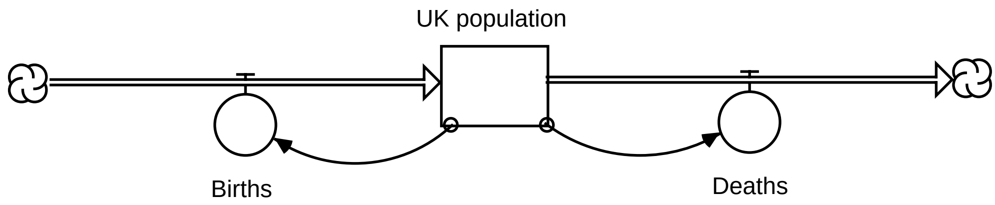

Fig. 2.1g.

Connectors show that the number of births and deaths depend on total population numbers.

Likewise the higher the population the more people die. Note that the details of the relationship between the rate of births and the population number are hidden in this approach, although it would have to be spelled out mathematically for a simulation to proceed. As we will see in section 2.2 it is possible to understand the behaviour of the model without the details of this relationship.

For the church growth model the number of converts may depend on how many unbelievers and how many believers there are:

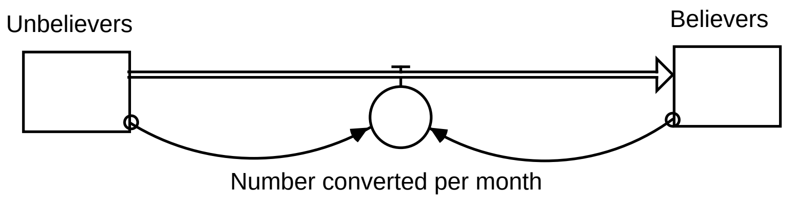

Fig. 2.1h. The number of conversions depends on the numbers of believers and unbelievers.

Essentially the more unbelievers there are the more conversions result in a month. under what circumstances this statement may be true will be discussed in chapter 3.

2.1.4 Converters

The connector in Fig. 2.1g showed that the number of births depended on the number in the population. In fact the greater the population the more people are born in a given month. The reason why the number of births is higher when the population is higher is because the *birth rate* is constant. The birth rate, the number of people born per person or per family, is a fixed quantity over short periods of time. Fixed quantities are represented by converters.

Different countries would have a different fixed birth rate. Deaths are treated in a similar fashion. This leads to a simple model of a growing or declining, population:

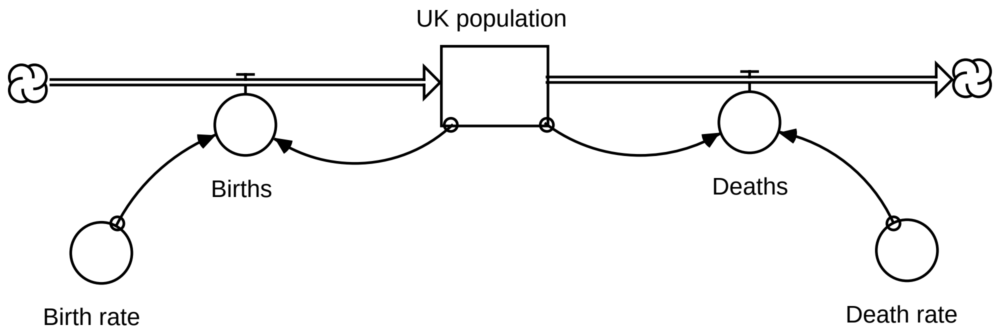

Fig. 2.1i. A model of UK population with births and deaths

Of course the birth and death rates of the UK population are not constant, but they vary due to effects not included in this model. Thus, to a first approximation, they are assumed constant. Such a model should give reasonable predictions over a twenty year period. More importantly it can give predictions indefinitely, given that the rates remain the same. Thus the behaviour of a population with constant birth and death rates can be thoroughly analysed.

Converters can themselves vary because they depend directly or indirectly on stocks. Consider The UK population split up into adults and children. Births produce children whose numbers are depleted by becoming adults (assuming no infant mortality). Adults increase by children growing up but decrease due to deaths. The total number in the population is given by a converter, and is variable as it depends on the number of adults and children:

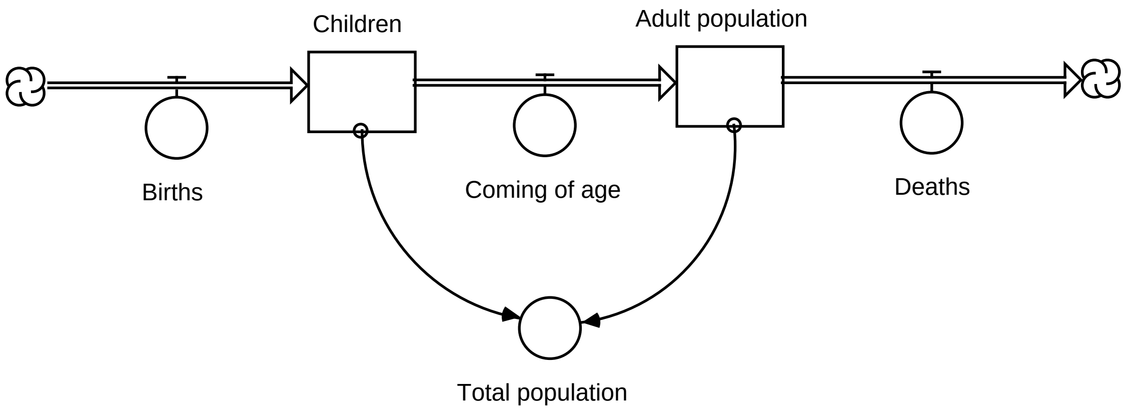

Fig. 2.1j. Converter (Total population) derived from other quantities

Thus converters either represent fixed quantities (constants) or represent variable quantities which are derived from other quantities, either stocks or other converters. Unlike stocks they do not represent fundamental variables Stocks change but depend on no other quantities.

2.1.5 Controllable Converters or Parameters

Some of the converters do not depend on any other converters, flow, or stocks in the model. This are parameters which need to be set before the model is run. Different values for these will give different results to the model

For example in the UK population of figure 2.1i, there are three controllable converters. The birth rate, death rate and the initial value of the UK population. The latter converter is not explicitly drawn on the diagram, although it would need to be present for the simulation of the model to run. Thus the UK population of figure 2.1i is a three parameter model.

2.1.6 Dynamic System

The combination of all these building blocks is a the *system*. It is a *dynamic* system because the stocks, rates and converters may change over time. The values of these quantities at any one time are referred to as the *state* of the system.

The dynamic system can be simulated using appropriate software, in this case Stella (also called Ithink). Because the simulation is a model of the real world system it is also called a dynamic model. The model can be investigated by graphing the values of the stocks, flows and converters over time, and by varying the values of the fixed converters and examining the effects.

2.1.7 Stability

If after a length of time the values of the stocks in the system become constant, i.e. they stop changing, the dynamical system is said to be stable. If a stock has only one flow this will happen when the flow rate becomes zero. Thus in figure 2.1h for believers to stop growing the number converted per month must be zero.

If there is an inflow and an outflow, as in figure 2.1i, the inflow must eventually equal the outflow if growth (or decline) is to cease. in this case births must equal deaths. It is possible that this situation can never occur. In this case there will always be change in the stock level.

For more complex situations growth ceases when the sum of all the inflows equals the sum of all the outflows. Like a tank of water for the level to remain constant the total amount of water from all the taps must be equal to the total amount flowing out from the drains.

2.2 Dynamic Processes

The model contains any number of stocks, flows, converters and connectors. As such models can get quite complex, as would be expected in a complex world. To help sift through this complexity it is possible to identify specific fundamental dynamic processes out of which more complex ones are built. These processes include the compounding, draining, and stock adjustment processes, which will be useful in church growth modelling.

2.2.1 Compounding Process

This is a growth process where the value of the stock reinforces the growth rate, that is the *inflow* is controlled by the stock itself:

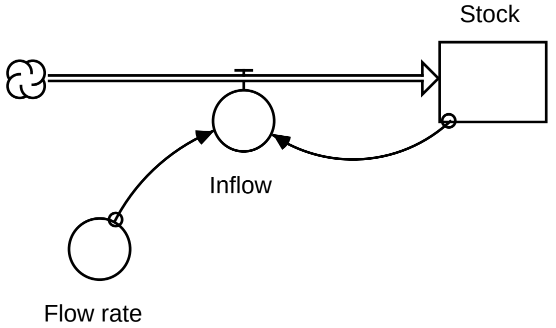

Fig. 2.2a. Compounding process

The inflow is the product of two inputs, the stock and a flow rate, according to the rule:

Inflow = Stock X Flow rate.

(Equation 2.2a)

This is commonly referred to as exponential growth, as its mathematical solution is a growing exponential function. In this case the inflow increases the stock which in turn increases the inflow (opens the tap more). Thus the increases get multiplied or compounded. Note the curve in figure 2.2b gets steeper and steeper. It is a model of the birth process where the birth rate is constant. It is also a model of the compound interest that banks use to calculate interest on savings and loans.

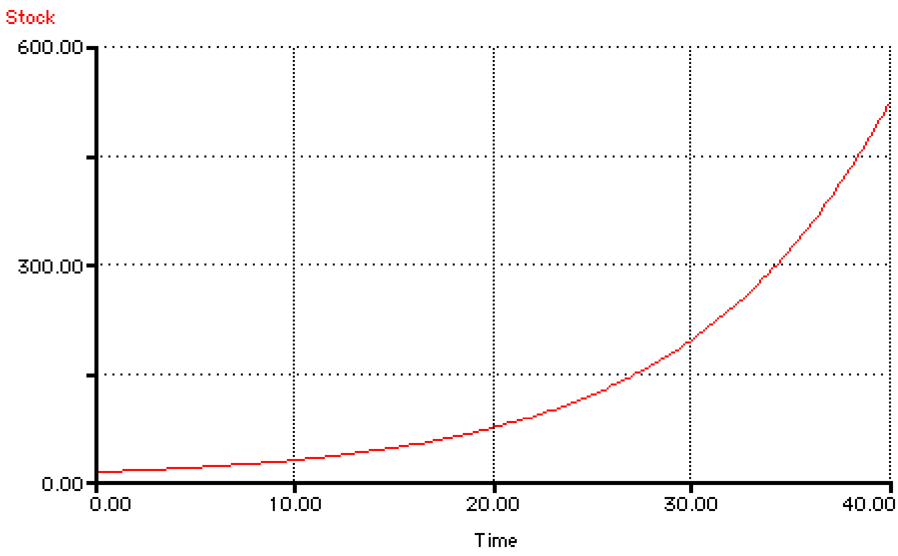

Fig. 2.2b. Exponential growth - result of a compounding process

The only situation where there can be no change in the stock is if the inflow is zero, i.e. the Stock is zero. So for non-zero stock levels there is always growth.

2.2.2 Draining Process

This is a decay process where the *outflow* is controlled by the stock:

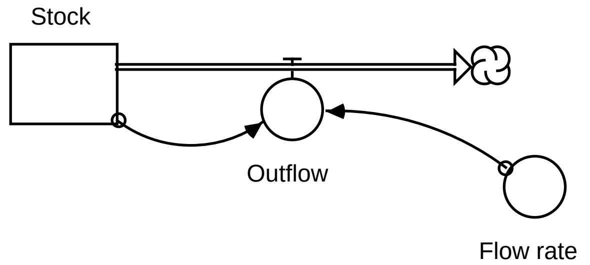

Fig 2.2c. Draining process

The outflow is the product of two inputs, the stock and a flow rate, according to the rule:

outflow = Stock X Flow rate

(Equation 2.2b)

This is commonly referred to as exponential decay, as its mathematical solution is a negative exponential. In this case the outflow decreases in time causing the stock to eventually level at zero. Growth stops when the outflow is zero which only happens when the stock is zero, equation 2.2b. Thus it is a decay targeted to zero. Note the curve in figure 2.2d gets lower but it is slowing down.

It is a model of the death process where the death rate remains constant. It is also a model of radioactive decay, indeed any decay process which has no memory of previous states, often called a Poisson process.

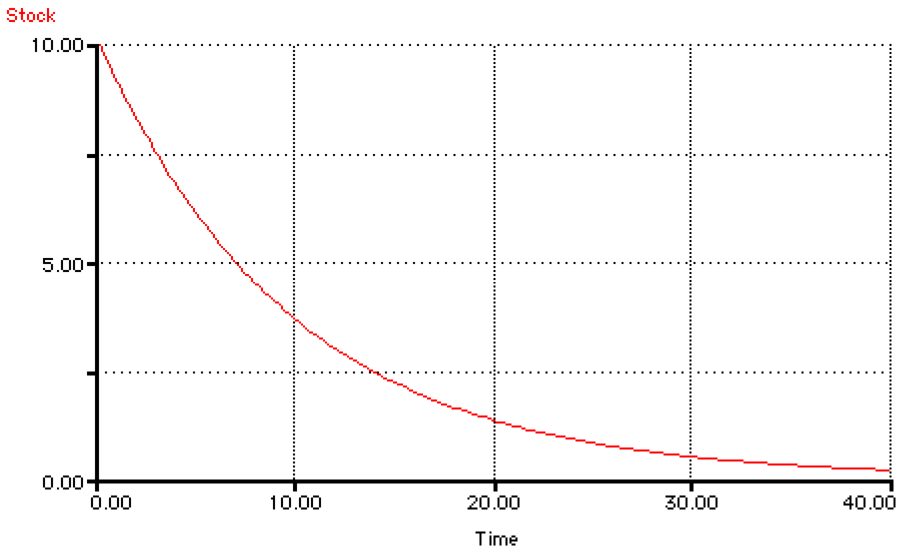

Fig. 2.2d. Exponential decay - result of the draining process.

2.2.3 Stock Adjustment Process

In this process the flow into or out of a stock is adjusted so that the stock can reach a target level:

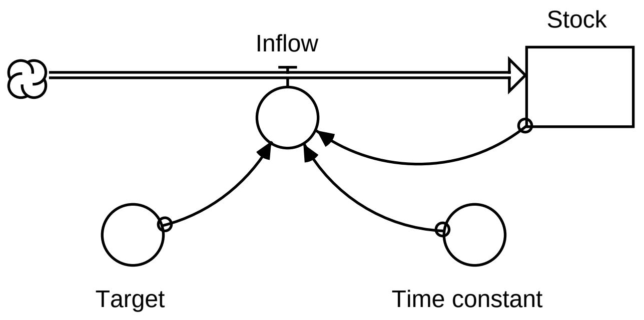

Fig. 2.2e. Stock adjustment process

Again this is an exponential limit model, where the stock tends to a fixed target level other than zero. In the example above the inflow decreases according to:

$$\text{Inflow} = \frac{\text{Target} - \text{Stock}}{\text{Time Constant}}$$

(Equation 2.2c)

where it is assumed that the level of the stock starts below the target. It is a model of the recruitment of staff where a target level of staff is required. In figure 2.2f the recruitment slows down the closer the stock (number of staff) gets to the target level. Growth ceases when the inflow is zero, i.e. Stock is equal to the target.

If flow is allowed in both directions it is a model of heat transfer where the stock represents the temperature of a body and the target is the background room temperature.

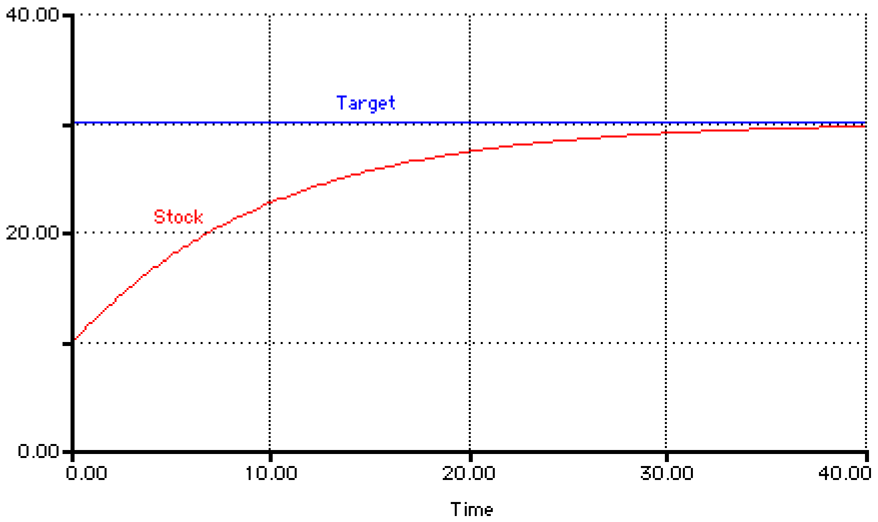

Fig 2.2f. Exponential limit - result of a stock adjustment process.

2.2.4 Other Processes

There are other fundamental dynamical processes that involve two or more stocks. However as these are not relevant to the following church growth models they will not be discussed. The reader is referred to more in-depth treatments of system dynamics

2.3 Causal Loops

2.3.1 Simple Causal Loop

It is possible to understand the general behaviour of a dynamic system without simulation. Conditions in a system give rise to some action and that action causes the conditions to change. A simple diagram, called a causal loop diagram (figure 2.3a) summarises this connection.

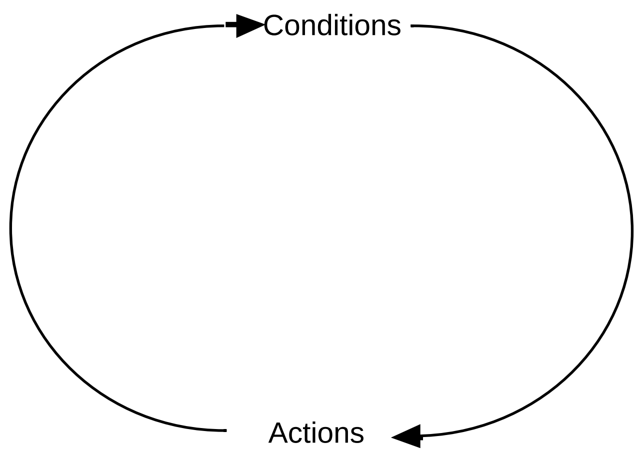

Fig. 2.3a. Simple causal loop diagram

The causal changes may be due to the *physical flow* of some quantity, or they may be due to *information links*. For example in the above stock adjustment model the recruitment rate causes staff numbers (the conditions) to increase. This is a physical link. However as the staff numbers get close to the target the recruitment rate (action) is reduced. This is a policy decision, a deliberate action, an example of an information link.

All systems models can be analysed with causal loop diagrams. The causal loops show the fundamental structure of a system, without being burdened by the details. Often, if a system is not behaving as it should, the solution is easier to see at the causal loop level than at the modelling level. Often it is a change of structure that is required such as an extra loop or the removal of a loop.

It is also possible to start the modelling process off at the causal loop level, and only move to the dynamic model once the structure is understood. This is particularly useful when the system contains "soft" variables, i.e. ones that are not easily quantified such as emotion, stress, desire etc. The population modelling in this paper will only contain "hard" variables based on population number - which are easily quantified. Thus the alternative approach of constructing the dynamical model first and then analysing it with causal loop diagrams will be taken.

Starting with the dynamic model, rather than the causal loop, will also avoid one of the common pitfalls of causal loop diagrams - the failure to distinguish between information links (connectors) and rate-to-level links (flows), as described by Richardson 1997. There is also a failure to distinguish between stocks (the system variables) and converters (the derived quantities. The type of link and the type of quantity effects the dynamics of the system and, following a number of authors, this paper will preserve that distinction in the causal loop diagram.

Some of the basic dynamical processes can be analysed with causal loops. The two fundamental loops are the reinforcing loop and the balancing loop. All complex systems are a combination of these two loops.

2.3.2 Reinforcing Loop

The reinforcing loop is the fundamental process of growth. If the increase in conditions leads to an increase in the actions, and an increase in the actions leads to an increase in the conditions then, the effect traced all the way around the loop, causes further increase in the conditions. Because of this accumulated increase it is often called positive feedback.

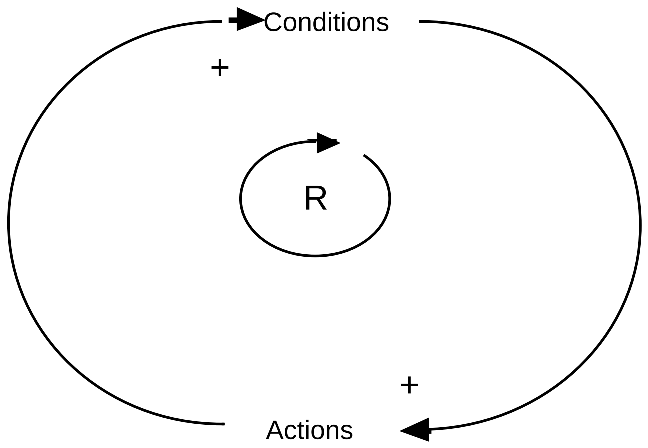

Fig. 2.3b. Reinforcing loop due to quantities reinforcing each other.

The compounding process is an example of a reinforcing loop:

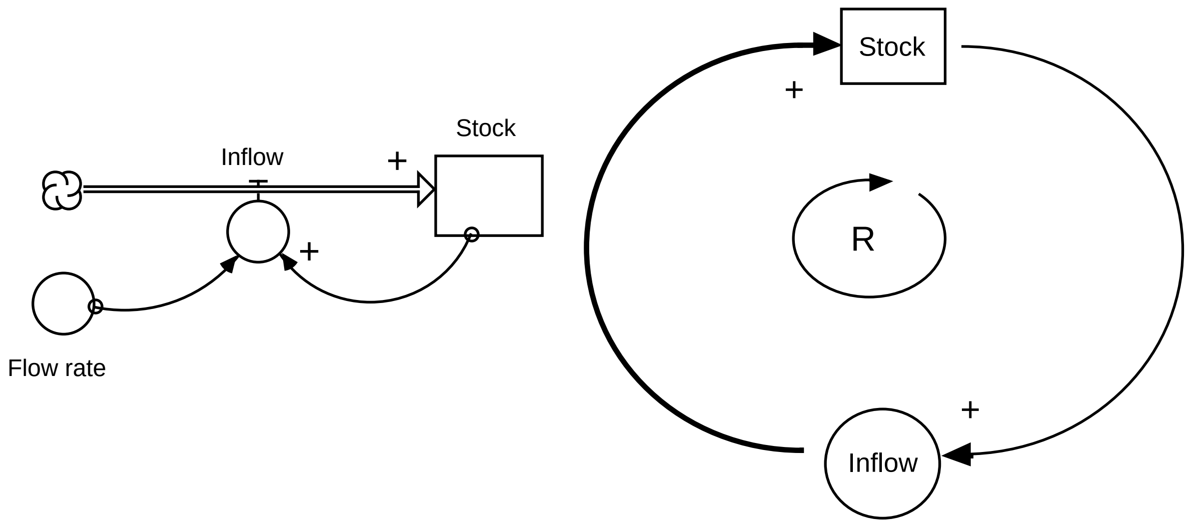

Fig. 2.3c. Compounding process as an example of a reinforcing loop

The thin line is an information link due to the connector from the stock to the inflow. The thick line is a rate-to-level link due to the physical flow into the stock from the inflow.

The flow rate is a constant and not part of the loop.

The plus sign refers to a positive effect of one quantity on the next. A positive effect in an <u>information link</u> can be seen by saying either of:

> An <u>increase</u> in *quantity 1* produces an <u>increase</u> in *quantity 2*
> An <u>decrease</u> in *quantity 1* produces a <u>decrease</u> in *quantity 2*.

For example an increase in the stock gives an increase in the inflow - thus its effect is positive or reinforcing.

A positive effect in a <u>physical flow</u> can be seen by saying:

> An <u>increase</u> in *quantity 1* produces a <u>faster</u> increase in *quantity 2*.

For example an increase in the inflow (opening the tap wider) will cause the stock to rise *faster*. The stock will always rise unless the inflow is zero.

Because each increases the other the net result back on the variable is positive, as it is reinforcing itself. It is thus called a <u>reinforcing loop</u> or, in more engineering terms, positive feedback. The birth process mentioned in sections 2.1.3 and 2.1.4 is a reinforcing loop. An increase in the population increases the number of births. This likewise causes a faster increase in the population.

It is also possible to have a link with a negative change, called an opposing link. If a loop has two opposing links it is also a reinforcing loop as the two negatives also make a positive:

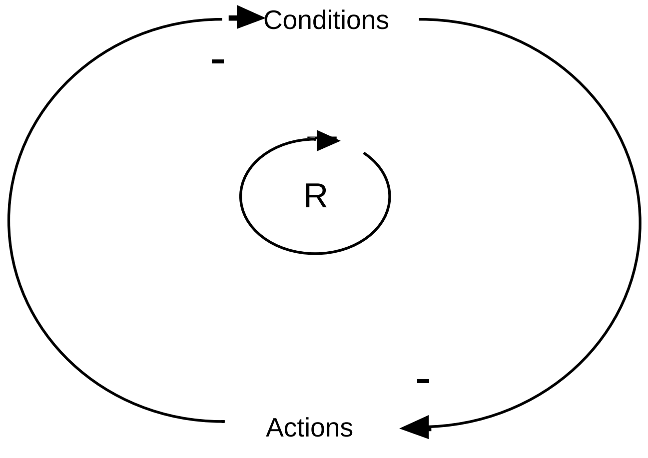

Fig. 2.3d. Reinforcing loop due to two opposing links

A negative link means that an increase in the conditions reduces the actions and a reduction in the actions increases the conditions. Thus the net effect back on the conditions is positive or reinforcing. However many items there are in a causal loop diagram, if there are an even number of opposing links then the overall loop will be reinforcing.

Reinforcing loops indicate instability. The feedback between a microphone and an amplifier, which results in a squeal that gets louder, is an example of a reinforcing loop.

2.3.3 Balancing Loops

In a balancing loop the net effect of all the changes around the loop back on the original variable is negative. It is thus called negative feedback. For example:

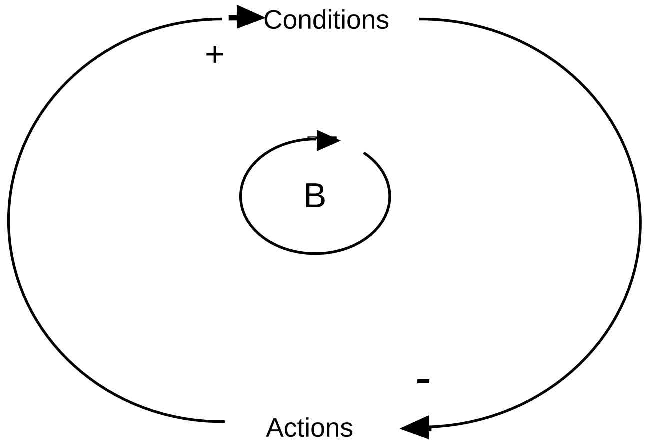

Fig. 2.3e. Balancing loop

Increasing the conditions leads to the actions being decreased. its decrease in actions causes the conditions to decrease. Thus the net effect of increasing the conditions is to bring it back down. The net effect is a balancing one.

The stock adjustment process is an example of a balancing loop. The inflow increases the stock. However, as the stock increases, and gets closer to the target, the inflow decreases:

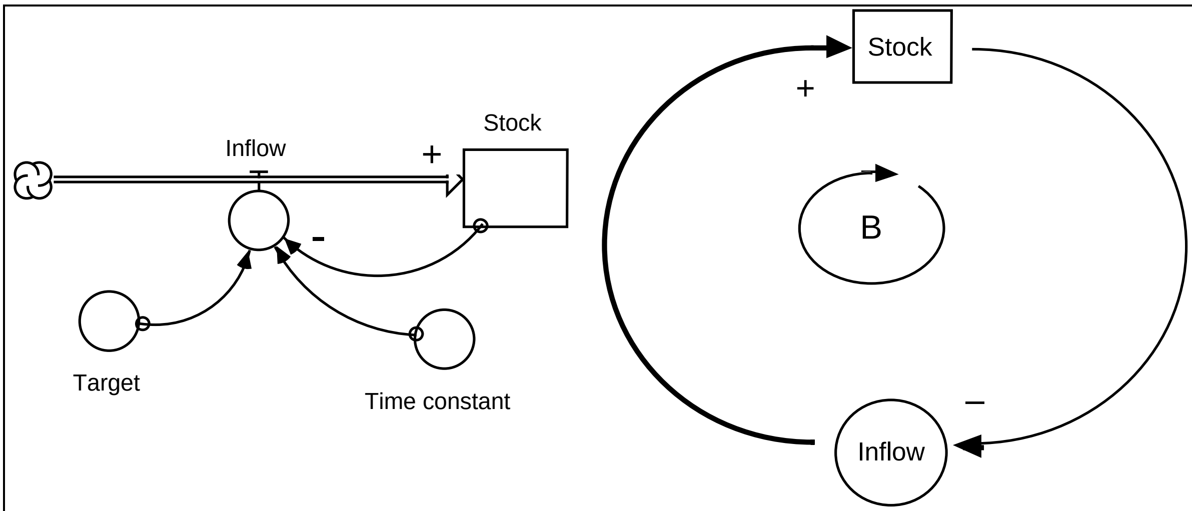

Fig. 2.3f. Stock adjustment process as an example of a balancing loop

A negative sign on an <u>information link</u> can be seen by saying:

> An <u>increase</u> in *quantity 1* produces a <u>decrease</u> in *quantity 2*.

In this case an increase in the stock causes the inflow to decrease. This was the process behind the staff recruitment model. An increase in staff (stock) leads to the recruitment rate (inflow) to decrease as target levels are nearer to being achieved.

It is also possible to have a negative link on a physical flow. This happens when the flow is an outflow. Reducing the stock through an outflow is a negative effect. The draining process is a an example of such a balancing loop:

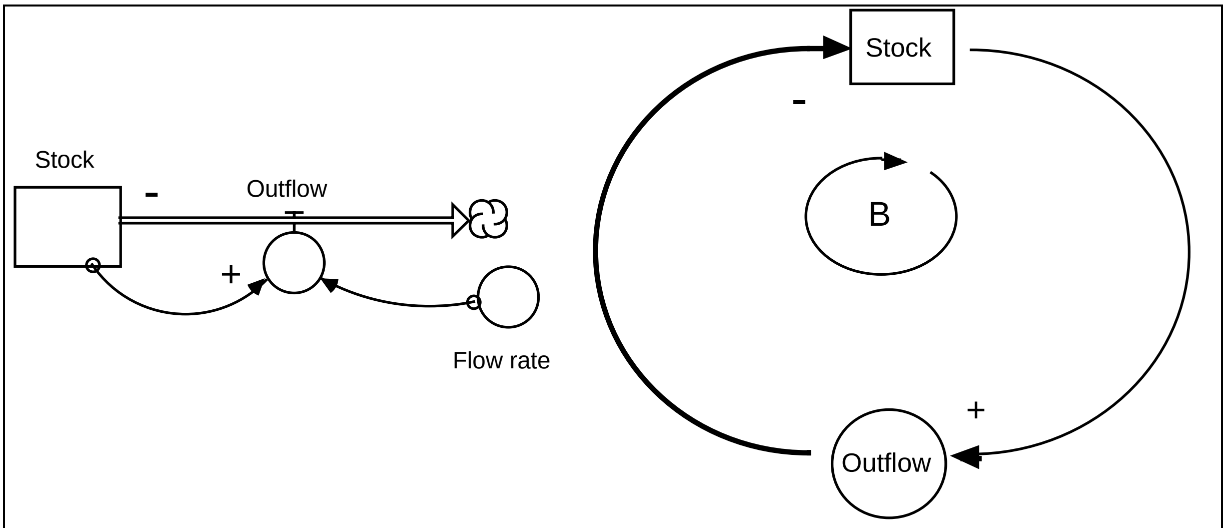

Fig. 2.3g. Draining process as an example of a balancing loop

A negative sign on a <u>physical flow</u> can be seen by saying:

An <u>increase</u> in *quantity 1* produces a <u>faster</u> decrease in *quantity 2*.

A more helpful way of saying this would be:

An <u>decrease</u> in *quantity 1* produces a <u>slower</u> decrease in *quantity 2*.

Thus decreasing the stock causes the outflow to decrease, the positive link. However the decrease in the outflow causes the stock to decrease <u>slower</u>, the negative link. The net effect is negative as the outflow slows down with decreasing stock. In this case it balances out to zero.

The death process in a population is a balancing loop. Balancing loops have a target. In the case of death the target is zero, but others are possible.

However many variables there are in a loop, if there are an odd number of negative changes, then it will be a balancing loop. Balancing loops indicate stability.

2.3.4 Loops In Complex Systems

The behaviour of a more complex system can be understood by combining together different balancing and reinforcing loops. As an example consider the logistic model of population growth. One example of this process is where a population grows in a fixed environment. Initially, when numbers are small and space is plentiful, the process is the compounding one reinforcing loop) - as the population increases the number of births, with a fixed growth rate, increases. However as the fraction of space taken by the population goes up the number of births decreases (balancing loop). This balancing loop becomes dominant and stability is achieved with the population being the maximum the space can support.

The dynamical model for this process is:

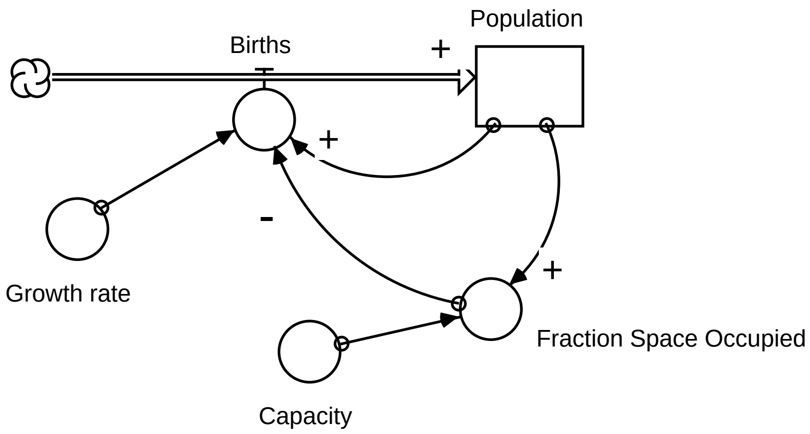

Fig. 2.3h. Logistic process - growth limited due to lack of space

This gives the S-shaped result for the population stock typical of logistic behaviour:

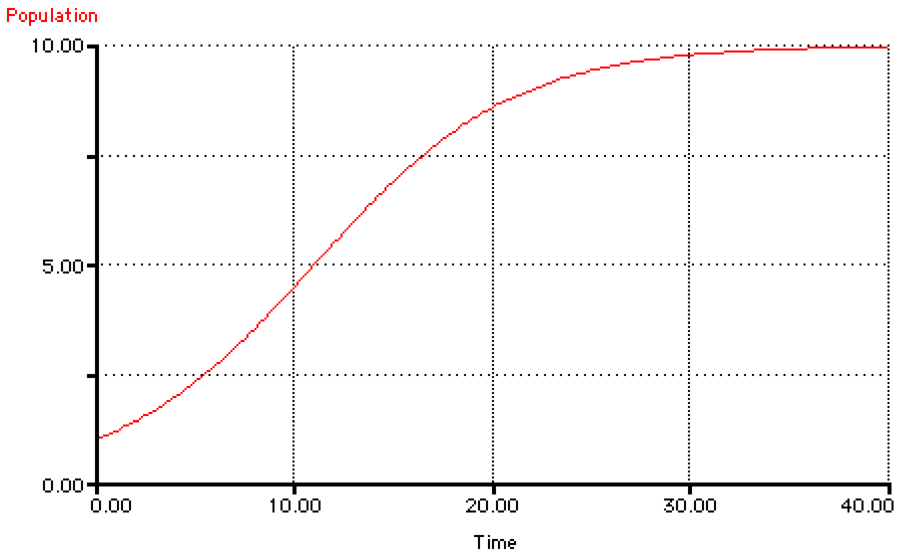

Fig. 2.3i. Graph of logistic process showing population reaching capacity.

The causal loop for this model shows the two loops:

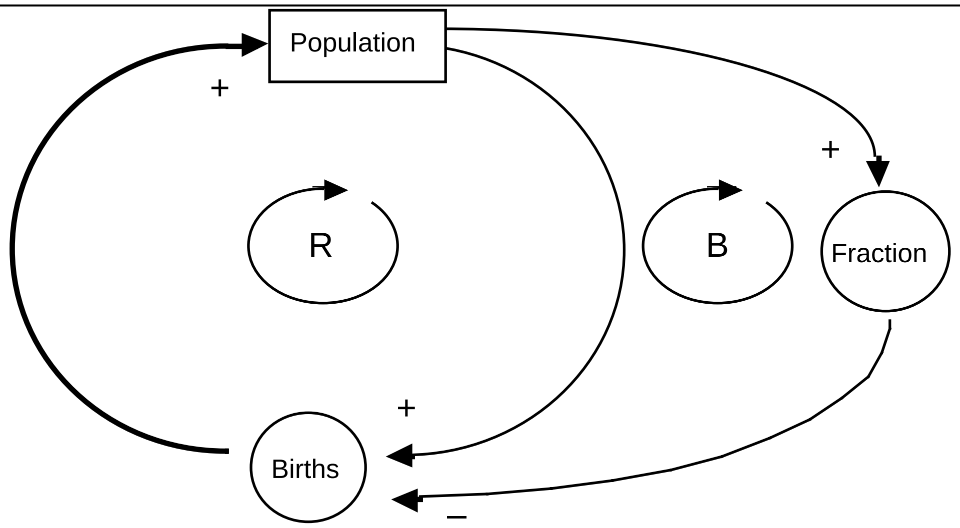

Fig. 2.3j. Causal loop diagram of the logistic model - shifting loop dominance.

The population is affected by two loops, one direct through births, and the other indirect through the fraction of space taken up by the population. The direct link is a reinforcing loop and gives growth, or "success". The balancing loop places limits on that growth. Thus this scenario is often called "limits to success", or “limits to growth”. It is an example of shifting loop dominance as first the reinforcing loop dominates and then the balancing loop dominates.

Note it is not clear from the above diagrams *how* and *when* the shift from one loop to another takes place. At this point more detailed modelling of the nature of the relationships is required. This would require mathematics.

Again determining the exact limit to the growth would require some mathematics. However some qualitative understanding can be gained by setting the only inflow, “Births”, equal to zero. The only converter that can do this is “Fraction Space Occupied”. Thus the limit is achieved when the fraction of space occupied by the population is at its maximum.

Other complex systems are modelled in similar ways. In fact there have been attempts to describe all systems in terms of a fixed number of archetypes. (Anderson & Kim 1998). There are very few truly different dynamical systems. However only the limits to growth one is relevant to the church growth model that follows.

3 Unlimited Enthusiasm Church Growth Model

3.1 Believers and Unbelievers

A church is a collection of people who have common religious beliefs, a common identity, and who meet together regularly usually for worship. As such they are an identifiable subgroup of society, including those who are like-minded, but largely excluding those who have no part of that religious life. Thus, at the simplest level, it is possible to split society up into two groups: those who belong to the church - called believers, and those who don't - called unbelievers.

Assumption 1.0: *The population is split up into two categories, believers and unbelievers*

3.2 Conversion Through Contact

In addition the believers have a common mission, which is to see unbelievers become believers. Effectively this means recruiting people into the church, called conversion. For simplicity such conversion will be deemed to be a one-way process, thus there is a drain from the unbelievers to the believers over time, with no other losses to the system. The model is thus a two stock model with transfer from unbelievers to believers:

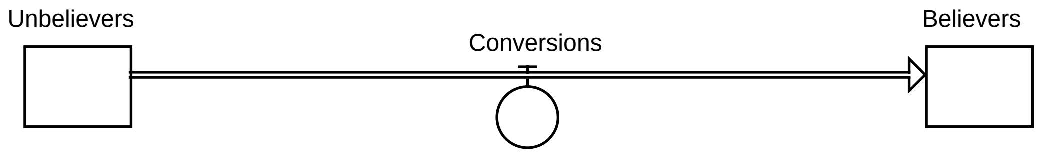

Fig. 3.2a The conversion process

where "Unbelievers" represents the number of unbelievers, and "Believers" represents the number of believers. The sum of the two is the total population which will be constant in this simplified system.

As was argued in Hayward 1999, the primary means of recruitment is through contact between an unbeliever and a believer. The contact itself may be the actual means of conversion, or it may be an invite to a meeting where the conversion occurs. Although many of these contacts occur between people who know each other, nevertheless it is possible that a believer's word of mouth testimony could inspire those they do not know to seek out the church and become a believer. This is a common feature during times of religious revival where, those effected by the revival are noticeably enthusiastic.

Assumption 2.0: *Recruitment to the Church is via contact between believers and unbelievers.*

Different people will have differing numbers of contacts with other people depending on lifestyle and family style. However as large numbers are being dealt with an average value can be taken as typical. Of course the number of actual contacts may be higher for believers than for unbelievers. A believers enthusiasm may cause them meet more people. Thus this average figure will be called the *number of contacts per believer per unit time*[3]. The average

[3] Unit time could be a year, or a month or in cases of rapid growth even a week.

number of contacts that unbelievers have with people is not relevant to the model, and it may well be different from the number of contacts that believers have.

Of course not all contacts will be with unbelievers. Thus the number of contacts with unbelievers will be affected by the probability they meet an unbeliever:

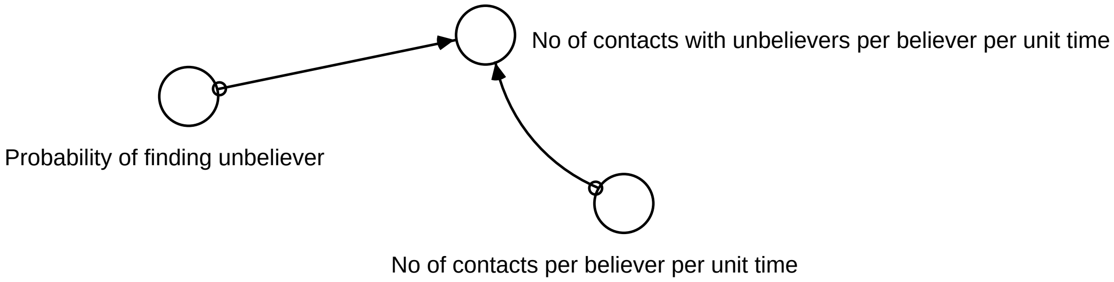

Fig. 3.2b Number of contacts with unbelievers for one believer

where

No. of contacts with unbelievers per believer per unit time =
Probability of finding unbeliever X No. of contacts per believer per unit time

(Equation 3.2a)

Assuming the believer does not deliberately seek out unbelievers, and has contacts uniformly mixed through the population, this probability will simply be the proportion of unbelievers in society. Thus

$$\text{Probability of finding unbeliever} = \frac{\text{Unbelievers}}{\text{Unbelievers} + \text{Believers}}$$

(Equation 3.2b)

This is often referred to as homogeneous mixing. It means the church has to be geographically spread, rather than confined to certain places. This would be true for much of the history of Christianity, apart from when it was introduced to a country. Even in its early years Christianity became spread throughout the Mediterranean world quickly due to good transportation between major cites.

Assumption 3.0: The different populations are homogeneously mixed

The actual number converted per believer per unit time will depend on the number of contacts with believers and the probability that contact leads to a conversion. Thus:

Actual No. converted per believer per unit time =
Probability of conversion X No. of contacts with unbelievers per believer per unit time

(Equation 3.2c)

In systems dynamics notation this is:

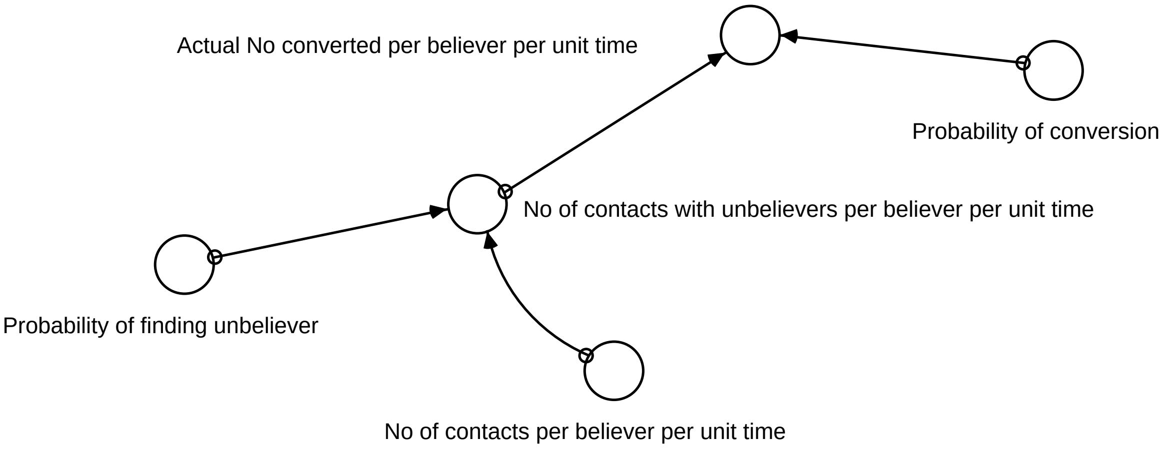

Fig. 3.2c Actual number converted through one believer

Thus the actual number of conversion will increase if either the probability of conversion increases, or the number of contacts increases. This appears to fit well with the mechanism of growth within the Christian church especially in times of revival. In such times believers have become more enthusiastic leading to considerably more contacts with people. At the same time conversion seem to become easier. Given a contact unbelievers are more likely to be converted. Revivals require this two-fold change: a change in a believer and in the unbeliever[4].

However there is a problem. No amount of increased contacts on its own will lead to any more conversions! It is this that sets spiritual conversion apart from a mere social phenomena. Conversion is ultimately in God's hands and it is impossible to set a probability of a conversion taking place given there is a contact. This probability, and the number of contacts per believer per unit time are not independent as both are used by God to achieve the conversion. So, although there is this two fold change required, there will be no increase in conversion unless the change in the unbeliever has taken place.

At the same time it is impossible to include an act of God in the model as a causal agent. A systems dynamics model is a model of the world, i.e. of creation. God is not part of his creation! More specifically every causal agent in these models has a prior cause, even if the model has not included that prior cause. For example a parameter such as the potential number converted per believer per unit time has a prior cause, but we have not modelled it here. However God has no cause - he is the only causal agent without a cause. Thus it would inappropriate to include such divine intervention on a model. It is better view God as upholding and guiding every feature of the process.

Thus although the model in figure 3.2c is perfectly adequate for social conversion, where people adopt a new fashion, lifestyle or join an organisation, it is liable to lead to confusion as a model of church growth. To model the spiritual conversion process at the heart of a growing church[5] the two converters: "No of contacts per believer per unit time" and "probability of conversion" need to be combined together as one parameter. Using equations 3.2a and 3.2c:

[4] See the definition given in Murray (1998) p23-24, (repeated in Hayward (2000) p.13 point 9), where this change in believer and unbeliever is explained further.
[5] The Christian church can also have socials converts as well as spiritual ones. See Hayward (2000) p.13 0.6.2.

Actual No. converted per believer per unit time =
Probability of conversion X Probability of finding unbeliever X
No. of contacts per believer per unit time

Giving:

Actual No. converted per believer per unit time =
Probability of finding unbeliever X
Potential No. converted per believer per unit time

(Equation 3.2d)

Where
Potential No. converted per believer per unit time =
Probability of conversion X No. of contacts per believer per unit time

Which in systems dynamics notation can be simplified to:

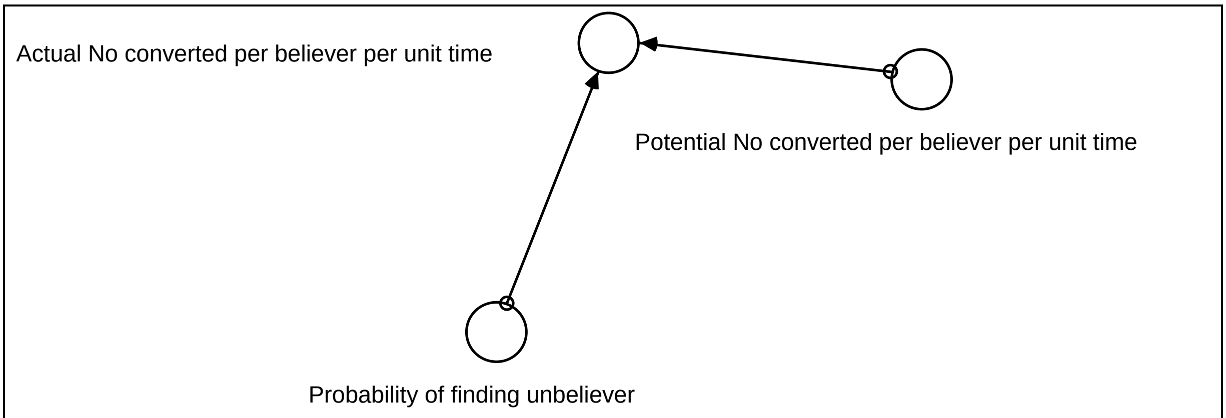

Fig. 3.2d Actual number converted for one believer for spiritual conversion

The single converter "Potential No. converted per believer" is a purely empirical quantity. It represents how many converts a believer would have been used in if the rest of the population were all unbelievers[6]. How this has been achieved due to changes in believer and unbelievers has been removed from the model. The model now focuses solely on the growth of the church regardless of the mechanisms of the conversion. This converter is also called the conversion potential of the believer, and is of course an average figure over all believers.

The conversion rate in figure 3.2a (the flow rate *conversions*) is proportional to the actual number converted per believer per unit time. As there are many such believers it is also proportional to the number of unbelievers. Thus the unlimited enthusiasm model of church growth is described by the stack and flow diagram in figure 3.2e.

[6] This was called $n_i(0)$ in Hayward (1999). A similar approach is used by Anderson and May (1987) in the spread of epidemics.

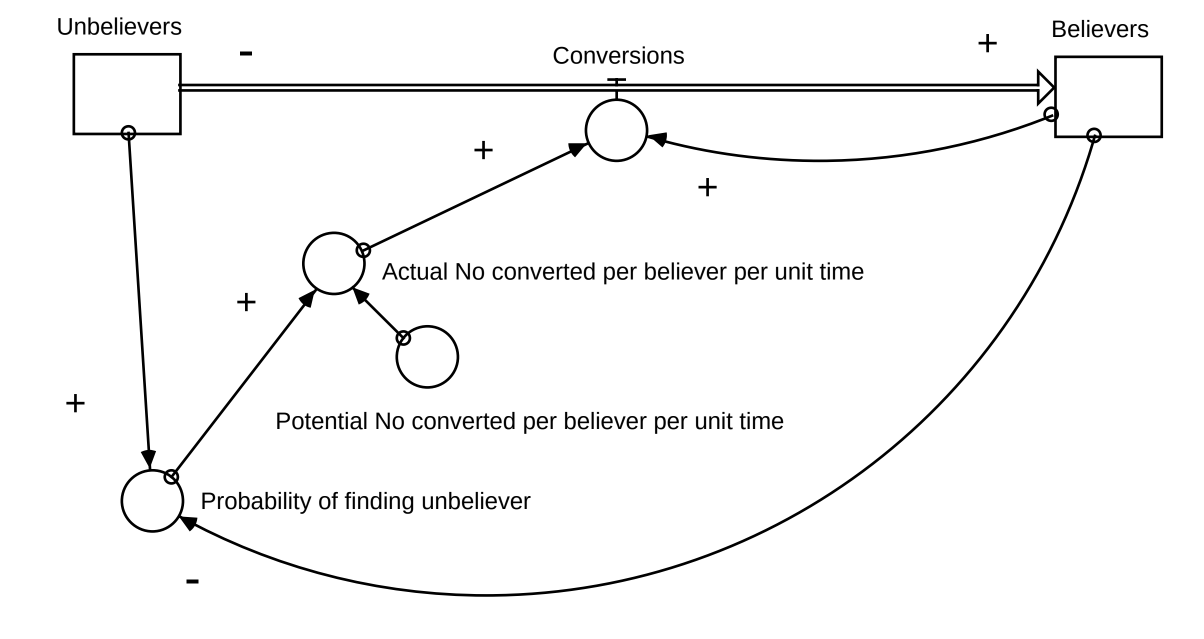

Fig 3.2e Unlimited enthusiasm model of church growth with information links

3.3 Further Assumptions

In this simplified model some other assumption have been implicitly made.

1. Believers do not become unbelievers again by any process.

   Assumption 4.0 Believers always remain believers.

   Thus there is no reversion - believers giving up the faith for internal reasons. Neither is there any persecution - believers giving up the faith due to fear, or the church being a threat to society. Likewise there are no alternative religions, or anti-religious groups seeking converts from the church.

2. Two crucial assumptions about the conversion process have been made. The believers do not cease trying to recruit unbelievers. Thus they do not run out of evangelistic zeal, or run out of unbelieving contacts before the pool of unbelievers is exhausted.

   Assumption 5.0 Believers continue to recruit in the same way indefinitely.

   Thus the believers enthusiasm is unlimited in time. This gives the name of the model Unlimited Enthusiasm Model of Church Growth.

At the same time unbelievers do not get hardened to the message from unbelievers or isolate themselves from the possibility of conversion. Thus:

   Assumption 6.0 Unbelievers continue to get recruited in the same way indefinitely

3. Over a period of time all human populations have births and deaths. However if that period is short, say less than 5 years, then these can be ignored. The results obtained without births and deaths will give the approximate behaviour of the system. Thus

Assumption 7.0: *There are no births and deaths within the system*

All these assumptions will be relaxed in later sections of the report.

3.4 Implications of the Unlimited Enthusiasm Model

Figure 3.2e also contains the signs of the causal links between the different elements. Note all the positive links from unbelievers to conversions. The more unbelievers there are the greater the probability of contacting one. The greater this probability the greater the actual number converted per believer. Thus the number of conversion is greater.

However the number of conversions is also greater if there are more believers. The number of conversions in a given time period depends on both the numbers of believers and unbelievers. This is the mass action principle originally proposed by Hamer (1906) in the spread of infectious diseases which is a fundamental feature of epidemic modelling. thus the form of this model is a simplified epidemic model where the infectious people remain infectious. There are three implications of this:

1. The number converted through one believer is <u>constant</u>. This encapsulates assumption 5.0 and 6.0.

   Thus on average believers always have the same conversion potential. Clearly this isn't necessarily true. The conversion potential depends on factors within the believer, which have been assumed constant (5.0). They may become more enthusiastic and make more contacts. They may explain the message more effectively. In times of religious revival a marked change comes over some believers causing them to be catalyst in the conversion of many people in a short time. Theologically this is explained as one of the results of an outpouring of the Holy Spirit. In social conversions “believers” may have some similar change of behaviour. However in spiritual conversion this baptism of fire, as it is sometimes referred to as, is different due to its intensity, effectiveness and change in the life of the believer. By taking the number of converted through one believer constant it has been assumed that no such change in behaviour has taken place, i.e. God continues to treat believers in the same way.

   The recruitment factor also depends on factors concerning the unbeliever. This is assumption 6. Thus in a sufficiently large enough group of people a certain amount of positive response will normally result. Of course if people are isolated from hearing the message this will not be true. Theologically it also assumes that God deals with different sorts of people in the same way. Different groups do not have different likelihoods of being converted given they understand the message.

   With social conversion this will not necessarily be true. Some groups of people may be more stubborn than others making recruitment less likely. But for spiritual conversion there is no will, however resistant, that cannot be softened, and no-one who can be changed unless God changes them. Thus again assumption 6 encapsulates the notion that God continues to act in the same way, this time with regard to the unbelievers he is converting.

2. Doubling the number of believers will double the number of conversion in a given time period.

   This will be the result of there being more contacts between believers and unbelievers in the same time period. For higher contact to result in more conversions it has been assumed that at any time the bulk of the unbelieving population have not heard enough of the message to be able to become a believer. Now more are hearing, the same proportion of a larger number are being converted. Hence more converts

3. Doubling the number of unbelievers will double the number of conversions in a given time period.

   For this to be true the believers need to be mixed throughout the unbelieving population - the homogeneous mixing of assumption 3. Believers also must not be at the limit of the number of contacts they can possibly make in a fixed time period. Thus more unbelievers will again lead to more contacts. Effectively we have assumed:

   Assumption 8.0: *There is no limit to the number of influential contacts a person can have in a given time period.*

   If this assumption is changed the result is the fixed contacts model mention in Hayward 1999. This will not be pursed further in this report.

3.5 Analysis

3.5.1 General Considerations

If the population of unbelievers is large in comparison to the believers, then growth of the church is initially exponential. At some point however the growth slows down and eventually ceases when the whole of the population has been converted. This occurs because the efforts of the believers are increasingly wasted on other believers as the number of potential converts declines. In the early years a single believer may know, or come into contact, with many believers, but as the church grows and the pool of unbelievers declines that believer will know fewer unbelievers.

This can be seen be examining the causal loop diagram for the model (figure 3.5a). This model is another example of shifting loop dominance and results in the S-shaped, or logistic, growth of the believing population. The sample simulation Fig 3.5b shows this logistic curve for unbelievers. Note that the unbelievers decrease over time following an inverse logistic curve, as the sum of believers and unbelievers remains constant.

The initial exponential growth is due to the reinforcing loop between believers and conversions. Increasing the number of believers increases the number of conversions thus increasing believers

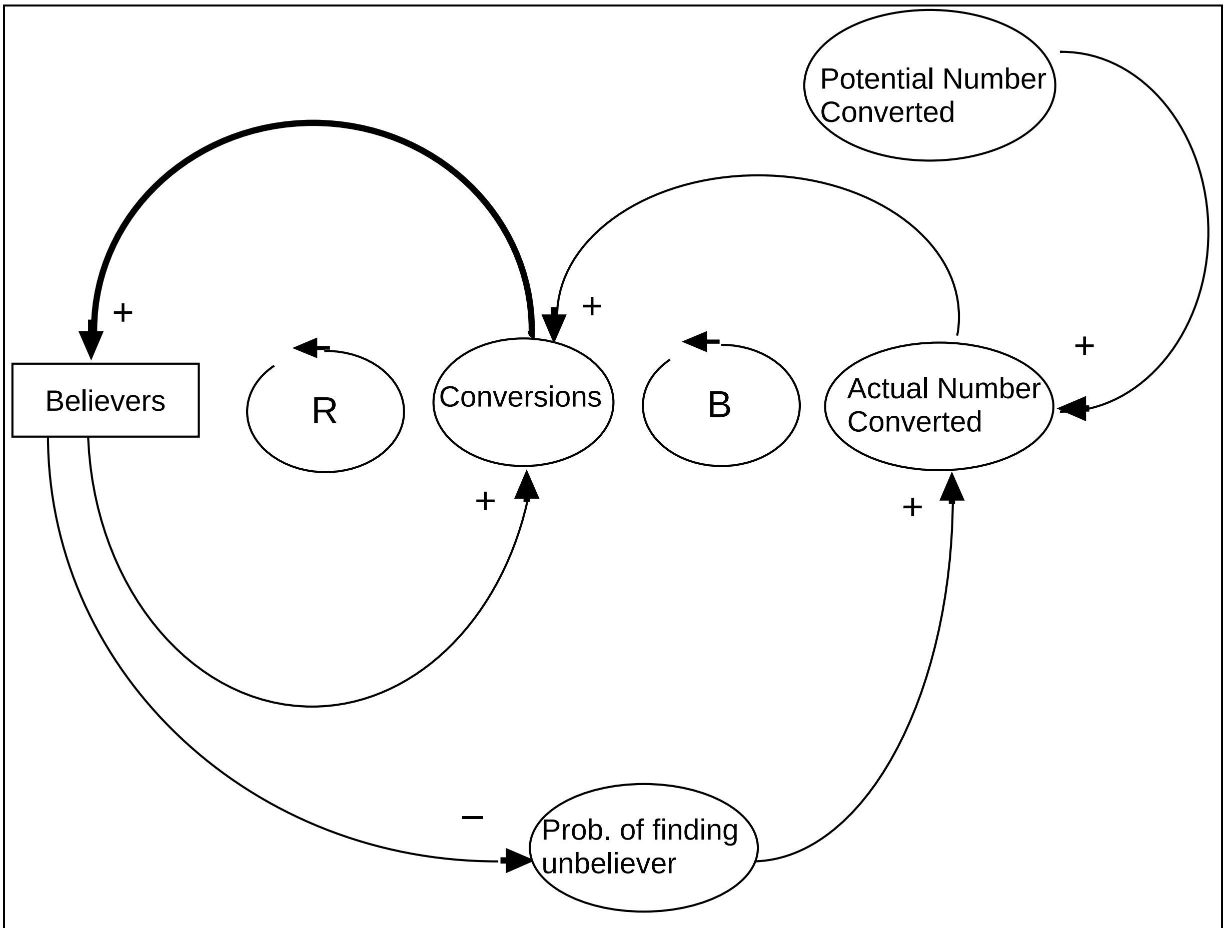

Fig. 3.5a. Causal loop diagram for believers in the unlimited enthusiasm model

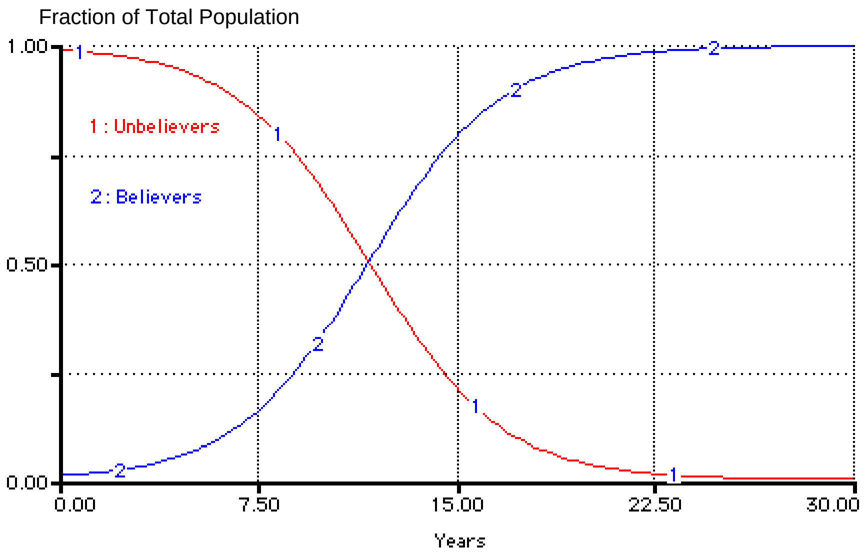

Fig. 3.5b Growth of the church through conversion with unlimited enthusiasm only

Once the number of believers gets large enough the balancing loop starts to dominate. Increasing the number of believers decreases the probability of finding an unbeliever, due to homogeneous mixing. Thus the actual number converted per believer and conversion rate also decrease, thus slowing down the growth of believers. However the whole population gets converted. The target of the balancing loop is set by the fact that growth stops when all rates are zero. ”conversions” is the only rate. For it to be zero the probability of finding a believer must be zero. From equation 3.2b this only happens when the number of unbelievers is zero. thus believers must be the total population.

The unbelievers follow the draining process of figure 2.3.2b with a single balancing loop (figure 3.5c). As seen above its target is zero.

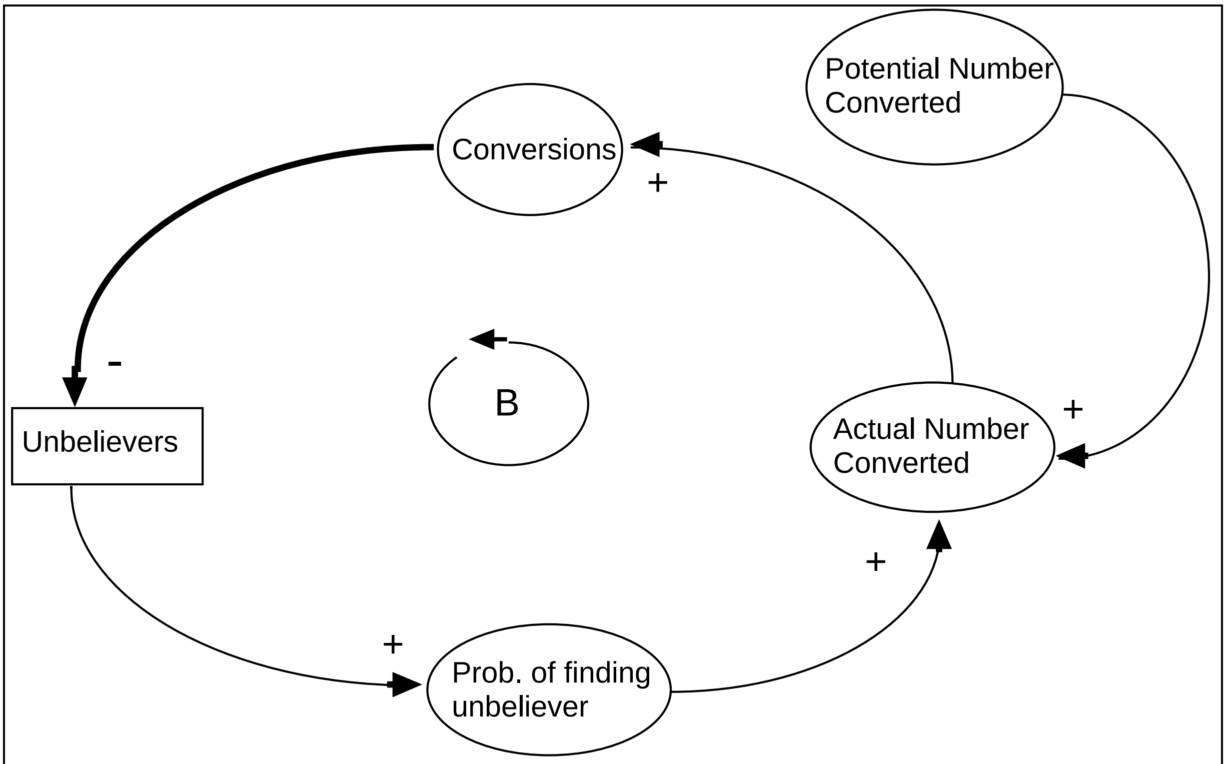

Fig. 3.5c. Causal loop diagram for unbelievers in the unlimited enthusiasm model

In mathematical terms this model is sometimes referred to as the simple epidemic model (Hayward 1999). However as the model is presented here will be called the unlimited enthusiasm church growth model. It can be shown mathematically that the model results is the logistic equation (Hayward 1999), where the logistic limit is the total number of people in the population. the model is often used for the dispersal of innovations through a population, the so-called Fisher-Pry models (Kumar & Kumar 1992). A similar model was used by Coleman (1964), and Bartholomew (1982) for the diffusion of social phenomena.

The time taken for the population to be converted depends on the initial numbers of believers and unbelievers, and also on the potential number of converts per believer per unit time. In this simulation the initial percentage of believers was 1%, and the number of people converted per believer per year was 0.4. That is each believer has had two contacts in five years that has led to a conversion. This results in the conversion of 99% of the population in just over 20 years. Such dramatic growth seems surprising from such a small conversion potential. The reason is that each new convert also makes converts at the same rate, and does so indefinitely. To simulate more realistic church growth these assumptions will need to be challenged.

3.5.2 Historical Considerations

This S-shaped growth is typical of the type of growth seen in revivals. A point comes when the growth of the church explodes because there are so many enthusiasts whose contacts are resulting in the conversion of unbelievers, many of whom also become enthusiasts. Such behaviour can be seen in the Welsh revival of 1904, where 100,000 people were converted on just over a year. Likewise the current revival in South America is seeing explosive growth. Nevertheless no revival, however powerful, has ever resulted in the conversion of a whole population. Thus some changes to the model are required.

The bulk of the conversions occur in the middle period of the growth. This can be seen by graphing the percentage converted against time (figure 3.5d). Thus early on in the growth there are few contacts, a thus few conversions, because the percentage of unbelievers is so small. Because growth is the main factor through which revivals are first noticed it can mean that a revival can be underway some time before it comes to the attention of the population at large. This was true of the early Christian church which remained small for the first 200 years. E.g. only 1% of the city of Rome was Christian about 250 AD. Yet by 300 AD the church was so widespread further persecution by the still Pagan state became impossible (Stark 1996). Similarly in Egypt estimates of the size of the Christian church based on inscriptions of Christian names shows similar slow then rapid growth (Bagnall1982)

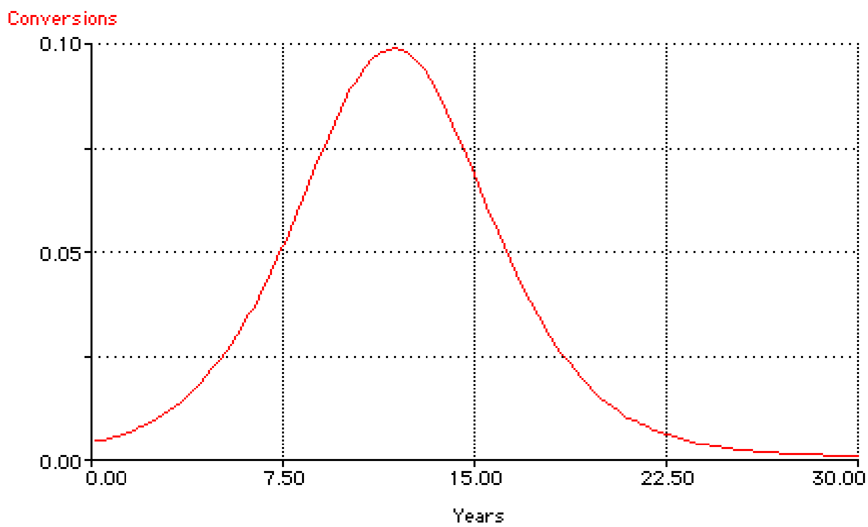

Fig. 3.5d. Percentage converted per year in the conversion model

This similar slow start can also be seen in the charismatic renewal through the 1970's which had not been really noticed except by those who were directly involved. However in the early eighties there was rapid growth in the number of independent charismatic fellowships - most of whom had been operating at a smaller level since the early days of the renewal. They only drew widespread attention when their growth had taken off after a number of years.

3.6 Conclusion

For the conversion model with unlimited enthusiasm:

- There are two parameters:
  - the potential number converted per believer per unit time;
  - the initial proportion of believers[7].
- Conversion leads to growth in the church that follows an S-shaped logistic curve.
- The whole population gets converted.
- The early period of growth is small and may go unnoticed. Low growth is not a sign that there is no work of God taking place.
- At some point very rapid growth occurs typical of many revivals

All further church growth models will be built upon this basic model by changing the assumptions given above.

[7] Technically there are three parameters including both the initial number of believers and the initial number of unbelievers. However as the total number is always constant then, as long as the only interest is in the proportion of society that believers make up, they can both be replaced by the initial proportion of believers. The initial proportion of unbelievers is just this 1 - initial proportion of believers. Note that Fig 3.5b just measures the fraction. Because of the principle of proportionality these results can be applied to any sized population.

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