Model Building with Soft Variables: A Case Study on Riots

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Abstract

A methodology for incorporating soft variables into system dynamics models is proposed. Building on previous research, the methodology uses a systematic assessment to identify soft variables, and concepts from software engineering to implement them. Data hiding is used to separate the units and scale of a soft variable from its effect on other model elements. By encapsulating the soft variable in a module with well-defined inputs and outputs, it can be used from a knowledge of its parameters alone, and not its internal construction, that is it is referentially transparent. The methodology is applied to an existing population model on riot growth, extending it to include soft variables whose scales are limited. The effects of the different soft variables on the populations are combined together using cognitive algebra. The extended model is compared to historical data and found to give a richer explanation of the riot dynamics than the original model. The paper is exploratory and intended to inspire further research.

Key Words: Social diffusion, soft variable, riot growth, data hiding, referential transparency, cognitive algebra

1 Introduction

The use of soft, or unquantified, variables is widespread in system dynamics, despite the difficulties associated with such variables. By their very nature soft variables, or soft concepts, are difficult or even impossible to measure; yet their inclusion in a model is often a matter of necessity, as they are known to be a part of a chain of cause and effect. For example in a public riot the size of that riot can encourage feelings of enthusiasm on the part of the rioters, encouraging them to recruit more to the cause. The size of the riot is easy to enumerate, but the enthusiasm of the riot is much harder to quantify. The temptation is to leave candidates for soft variables out of a model, as calibration would be difficult. However the much quoted comments of Jay Forrester (1961, p. 57) should encourage anyone to resist such a temptation: "To omit such variables is equivalent to saying they have zero effect – probably the only value that is known to be wrong!" Such an approach would be in the words of Sterman (2002) "a sure route to narrow model boundaries, biased results, and policy resistance". The purpose of a model is to explain

behaviour by theory and hypotheses, and thus if the concept is part of that explanation it needs to be included in the model, however difficult it is to quantify.
System dynamics as a modelling methodology is not just an explanation of cause and effect, it is also a simulation tool. As such there comes a point where a soft concept has to be turned into a variable with numbers attached to enable a simulation, however "unquantifiable" the concept may be. Indeed once the word "variable" is used, it is almost implied that a numeric equivalent of the concept has been constructed. A methodology as to how such concepts can be turned into simulation variables is more problematic. In his review of the problems of qualitative modelling, Coyle (2000) raised research questions with regard to soft variables, suggesting the need for procedures to understand their identification, measurement, construction and validation. In reviewing the work of authors both previous and subsequent to Coyle (2000), it will be convenient to group them under those four headings.

A. Identification and Nature of Soft Variables

From a sociological perspective, Jacobsen & Bronson (1987) gave very helpful guidelines for handling sociological variables, which can be carried over to soft variables in system dynamics (Levine, 2000). Such a variable must be:

(i) Reliable – it must have units of measurement and have a meaning that can be agreed by all;
(ii) Realistic – it must correspond to some concept in the real world;
(iii) Have face validity – it can be reasonably substituted by an indirect indicator, that is a similar concept with a better defined means of measurement and thus closer to a variable.

B. Scales and Units of Soft Variables

Levine (1983) noted that psychologists deal with soft variables and have used correlation analysis to obtain stable and relevant measures of "fuzzy" concepts such as anxiety and cognitive complexity. Levine (2000) also showed how psychological concepts could be modelled using system dynamics, highlighting the conceptual problems of allowing the soft variable to go infinite. Instead he proposed limiting the scale to 0 to 100, to allow the use of percentage points, using a limits-to-growth balancing loop to control the upper bound of the variable.

Nuthmann (1994) discussed four different scales of variables: *nominal* – used to describe characteristics that have no numerical value such as gender, ethnicity, which unordered descriptive values; *ordinal* – where values can be place in rank order but without a sense of inter-value distance; *interval* – where there is an additive inter-value distance; and *ratio* where that distance is multiplicative. He argued for the interval scale as the most natural. By contrast Levine (2000) puts a case for ratio scales in soft variables, including how they can be related to a measurement of the variable on an interval scale. In either case it is clear that soft variables have a sense of order, i.e. they are at least ordinal.

C. Construction of Soft Variables

Levine & Doyle (2002) discussed types of soft variables using a variety of generic structures. Initially looking for social archetypes they decided that a social psychological molecule was more appropriate as they were attempting to "capture the dynamics of small bits of social processes", rather than a generic behaviour pattern.

From this perspective a soft variable is no longer just a stock but includes appropriate converters and connectors in order for it to respond to inputs in a particular way. This suggests that a modular approach to soft variables could be developed.

Nuthmann (1994) discussed combinations of soft variables based on the cognitive algebra of Anderson (1981). He identified three: addition, averaging and multiplication. McLucas (2003) took this further, developing a method of weighted variable combinations according to their perceived importance (c.f. Sterman, 2000, Ch. 13). Thus if a modular approach to soft variable construction is taken, thought is needed as to how the outputs of such modules are combined.

D. Validation of Models with Soft Variables

Sterman (2002) stated that soft variables should be used in simulation when necessary, providing the model is properly validated, including historical and statistical fits, but not exclusively. He suggested modellers need to ask why soft concepts used in modelling have not been measured, and gives examples of soft concepts for which measures, albeit imperfect, have been obtained once their importance was established. Roy & Mohapatra (2003) suggested a method of validating models with soft variables using structural equation modelling.

The purpose of this paper is to put forward a possible methodology by which soft variables can be incorporated into a system dynamics model, using some of the insights referenced above. The method is divided into three main stages: the identification, the construction and the use of soft variables. *In particular it is proposed that soft variables are understood by their use, rather than by their internal construction*; *and that their measure is dealt with separately to their effect*. To achieve this understanding a modular approach to their construction is taken, using ideas from software engineering.

The proposed method is both exploratory and tentative. It is intended to inspire discussion and further investigation rather than enumerate a definitive set of guidelines. A thorough discussion of model calibration and validation is beyond the scope of the paper, though some comments will be made in the discussion.

To illustrate the proposed method, an existing model on the dynamics of riot growth by Burbeck, Raine, & Stark (1978) is extended to include soft variables. The original model used only hard variables in the form of population numbers, being modelled by analogy with an SIR epidemic. However the problem description had causes that would have allowed for the use of soft variables, producing a model with a richer explanatory power.

2 Model Description

2.1 Original Riot Model

In 1978 Burbeck, Raine, & Stark put forward a mathematical model for the dynamics of the growth of a riot with specific application to the riots of Los Angeles 1965, Detroit 1967, and Washington DC 1968. The primary hypothesis was that rioters recruited other rioters from the population by word of mouth. It was further hypothesised that rioters left the riot after an average length of time either due to tiredness or arrest. As such the model had the same structure as that of an SIR epidemic model.

Although there was no direct information on the number of rioters over time, the authors had access to police, fire service and civil defence records that gave reports of incidents during the progress of each riot, recorded by time of day, which they compiled into a unified time series of reported events. They further assumed that a fixed percentage of rioters caused the damage, and were thus able to show that the reported incidents followed the expected pattern of the infected of the SIR model. The duration of each riot was about 5 days.

The model of Burbeck et. al. (1978) was entirely composed of quantified variables: potential rioters, the susceptibles; rioters, the infected; dropouts, the removed, and riot events, a normalised variable to represent the occurrence of reportable events. However they also referred to other factors that influenced the dynamics of the riot such as the media reports and the contagiousness of the rioters. The former they ignored and the latter they assumed was constant even though it would vary with the enthusiasm of the rioters and the strength of the cause. Additionally they discussed the level of sympathy among the potential rioters. Media reports, enthusiasm, sympathy and strength of cause are of course much harder to quantify, but an attempt at their inclusion in the model would make a useful illustrative test case for a possible methodology for modelling with soft variables.

The original model of Burbeck et. al. (1978) was written using differential equations. However it can be re-expressed as the equivalent system dynamics model, figure 1.

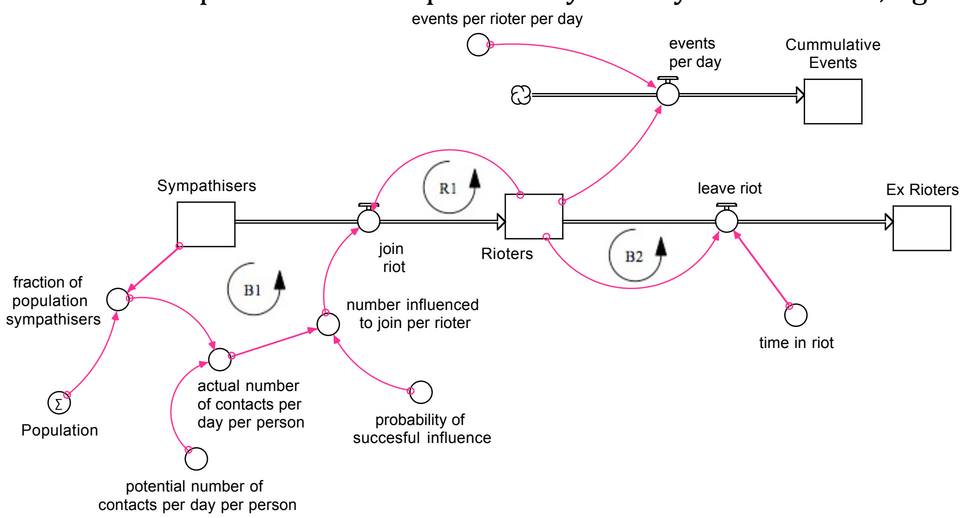

Figure 1: SIR Model of Riot Growth based on Burbeck, Raine, & Stark (1978)

The hypothesis that rioters recruit to the riot by word of mouth is expressed in the reinforcing loop *R1*. Their reduced effect on the sympathisers is expressed in the balancing loop *B1.* The original model considered a single leaving rate of rioters from the riot. The limited time rioters spend in riot activity, whether due to arrests or self action, is modelled by the single loop *B2*, reflecting the hypothesis of Burbeck et. al. (1978). The authors compared the reported events over time with *events per day* in the model showing that the model could reproduce the observed pattern for certain parameter values.

2.2 Model Extension

For the purpose of extending the Burbeck model to include soft variables, the following additional concepts and hypotheses are considered:

1. The <u>strength of the cause</u> which originally sparked the riot;
2. The <u>number of arrests</u> due to the action of the police. This is assumed to be the cumulative number rather than the arrest rate, as the impact of arrests on rioter enthusiasm is more likely to be accumulative over the short period of the riots being modelled;
3. The <u>enthusiasm of the rioters</u>. This enthusiasm will influence recruitment and retention of rioters. Enthusiasm will be generated through the size of the riot and the strength of the cause, but will be dampened according to the number of arrests;
4. <u>Media interest</u> which creates sympathy for the riot, and also will increase the effectiveness of recruitment to the riot. Media interest will increase as the number of reported events increases as such events measure both the size and seriousness of the riot;
5. The <u>sympathy of the non-riot population</u> towards the riot. That sympathy makes recruitment to the riot more likely and is in turn affected by the media interest.

These concepts lead to potential soft variables: *Strength of cause*, *enthusiasm of rioters*, *media interest*, *sympathy of non-rioters*, and *events*. Although *events* was a quantified variable in the original model, its definition will be reconsidered as a candidate for a soft variable.

3 Methodology

The methodology used to turn the concepts into soft variables is divided into three stages: the <u>identification</u> of the variable; the <u>construction</u> of the variable; and the <u>use</u> of the variable. It will be assumed that a causal loop diagram of the whole model has already been constructed so that the causes and effects of each potential soft variable have been determined.

3.1 Identification of Soft Variable

The aim is to assess whether the concept can be thought of as a numerical variable.

General Description

How is the concept informally described? This should give an initial assessment or description of the concept to be modelled and explain why it is needed. It can be helpful to describe its causes and effects as this may shed light on the description. This corresponds to the nominal variable definition of Jacobsen & Bronson, 1987.

Aggregation

Is the concept a single irreducible entity or is it a composite structure, a collection of entities? The purpose is to give reasons why the concept or entity may be modelled as a single variable or a number of variables. That is, is the entity one-dimensional or multi-dimensional? The principle of Occam's razor should be used, i.e. entities must not be multiplied beyond necessity.

Ordering

Does the concept have a clear ordering? The main purpose is to help justify why a single variable could be used. If a sense of ordering were ambiguous it may imply that a many variable, multi-dimensional model might be more appropriate. Identification of an ordinal scale is necessary for the concept to be a single variable (Nuthmann 1994).

Potential Measures

How could the concept be potentially measured? Although the measurement will not necessarily be built into the model this exercise helps clarify the variable.

At this stage it should be clear whether a concept is a candidate for a soft variable, i.e. it should have reliability, realism and face validity (Levine, 2000). It is not necessary yet to know how it will be modelled or used in a system dynamics model; that is the next stage. From now on the concept will be referred to as a variable.

3.2 Construction of Soft Variable

Levine (2000) distinguished the <u>use</u> of a soft variable in a model from its <u>measurement</u>. The latter could be seen as its effect on the observer, rather than on the model elements. Thus it is proposed that the soft variable model is constructed in a modular fashion such that its inner workings, including its measurement, are hidden from the other model variables in its cause and effect chain. In software engineering this concept is referred to as data hiding (Booch et. al., 2007) and enables modules to be tested prior to their inclusion in a larger model. Once incorporated into the system dynamics model the soft variable will be understood in terms of its input and output alone, a concept referred to as referential transparency (Bird and Wadler, 1988). Such a modular approach for a soft variable is an example of the social psychological molecules of Levine and Doyle (2002).

Thus the soft variable will be constructed so that both its numeric value and dimensions are hidden from connecting elements. Its internal numerical value is not relevant, nor is its unit of measure, only its response to stimuli and its effect on other variables. Thus the effect of a soft variable on other elements can be dimensionless, enabling the effects of different soft variables to be combined together without regard to their hidden units.

To complete the construction of the soft variable model the following stages are considered:

Scale

Does the soft variable have a minimum value? Does the variable have a maximum value? That is, are there limits on the scale of the variable? If the answer is yes in either case then the values of these limits are set. In the remainder of the paper discussion is confined to variables on a limited scale.

Units

Are there any suggested units of measure for the soft variable? A discussion of appropriate units naturally comes out of the previous discussion of scale. Although these units will be hidden from connecting elements, for the sake of internal dimensional consistency some units should be used. If no unit of measure is natural then an abstract unit should be defined.

Nature

Is the soft variable a stock, converter or flow? A consideration of units will help distinguish a stock from a flow, as will the photograph test (Sterman, 2000). A consideration of memory or delays can help distinguish a stock from a converter.

Inputs

What outside elements have an effect on the soft variable? I.e. what is the effect of its causes? If the decision is to model the soft variable with a stock then is its growth and decline mechanisms should be consider. It is suggested that endogenous mechanisms are tackled first.

Outputs

What outside elements does the soft variable affect? There must be at least one output otherwise the variable would not be required. Although different outside elements may be affected by the same internal element of the soft variable, the possibility of more than one type of output should be considered. The output should be constructed so that the internal units are hidden and that it follows any limits set on the scale. For the remainder of the paper it is assumed the soft variables have an output scale limited from 0 to 1.

As the identification of soft variables progresses it will be helpful to re-express the causal loop diagram into a modular diagram to help highlight the causes and effects of each soft variable.

3.3 Use of Soft Variable

Once the model for the soft variable has been constructed (and tested) consideration is given as to how it connects to the wider model. This will be subdivided into the effect of the soft variable and the way different soft variables need to be combined. Reference should be made to a modular version of the causal loop diagram.

Effect

How many different model elements does the soft variable affect? There will be a separate model for each effect of the soft variable, which will allow it to have different impacts on different elements. Essentially there is one effect model for each link from the soft variable. These models can be modularised. If the soft variable has any scaling then the scaling can be preserved in the effect.

Combinations

How are different soft variable effects combined before they influence another element? Popular combinations are multiplication, addition and weighted averages, which may be linear or non-linear (Sterman, 2000, Ch.13). Multiplication of effects is often assumed, which can be problematic if there are many such combinations as the lowest factor can dominate the result (Coyle, 2000). McLucas (2003) also warns of the dangers of more esoteric combinations and advises that combinations are kept as simple and natural as possible.

Following the work on cognitive algebra of Anderson (1981) and Nuthmann (1994), the number of combinations of two variables are limited to four, table 1, where it has been assumed the variables in question have been limited to a scale of 0 to 1.

Description	**Mathematical Construct**	**Formula** $f(x,y)$
Strict	Logical AND	xy
Strict Compromise	Harmonic Average	$\sqrt{xy}$
Lenient Compromise	Arithmetic Average	$\frac{x+y}{2}$
Lenient	Logical OR	$x+y-xy$

Table 1: Combinations of Soft Variable Effects $(0 \le x, y \le 1)$

The *strict* combination of soft variables *x* and *y* is the usual multiplication model, and is the equivalent of the logical AND. That is, if the effect of one soft variable is switched off, the other has no effect. If one is set to the maximum value of 1, the combined effect is the full range of the second variable. Other combinations are shown in figure 2. This case is referred to as *strict* as the output is less than the smallest input, $x < y \Rightarrow xy < x < y$. This can be thought of as the most pessimistic approach to combination, as the one soft variable restricts the effect of the other.

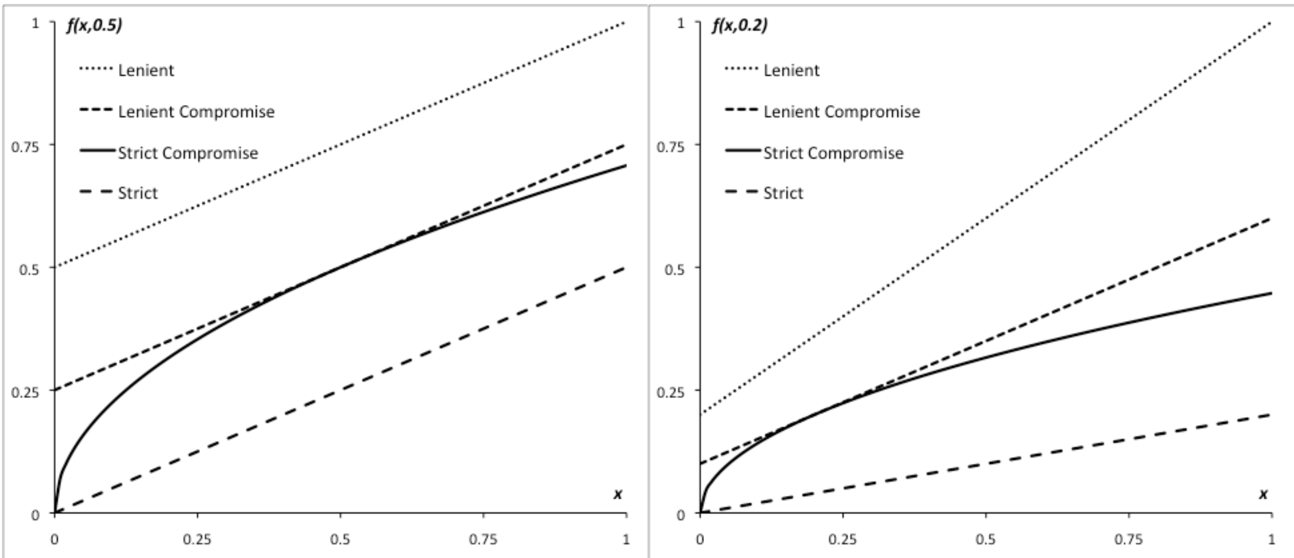

Figure 2: Cognitive Algebra. Output as a function of one input x with second input 0.5 and 0.2.

There are two models of a compromise position between the two variables. The *lenient compromise* combination is the familiar averaging. An un-weighted combination is used as it is assumed that the effects of the two variables are already on comparative scales. The level and response for two given inputs is higher than the strict case AND (figure 2). A more tentative suggestion is made for a *strict compromise* combination using the harmonic average. There will be a point where its response can match the lenient compromise, but at the extremes of one of the inputs being 0 the response matches the strict case (figure 2). The comments of McLucas (2003) about this combination being "an abuse of mathematical logic and system dynamics principles" are noted, but it is hoped that the use of the dimensionless <u>effects</u> of the soft variables with a clear logic-based methodology have avoided the issue. For both compromise combinations if the effects of the two soft variables are the same the output matches that effect, $f(x,x) = x$. In addition the output of both compromised combinations lies between the smallest and largest inputs $x < y \Rightarrow x < \sqrt{xy} < \frac{x+y}{2} < y$, which suggests the term compromise.

The final combination is based on the logical or cognitive OR, is referred to as the *lenient* combination, and can be thought of as the most optimistic case. If one soft variable is switched off the response follows the full range of the other variable (figure 2). If however one variable is at its maximum 1, the second variable can have no further effect. The response of OR is higher than all the other combinations for the same inputs, and is larger than its highest input $x < y \Rightarrow x < y < x + y - xy$. In this sense the one soft variable enhances the effect of the other.

For all combinations the scale of 0 to 1 is preserved, and thus do not have the scaling issues raised by Coyle (2000).

4 Model Construction

In this section the riot model of Burbeck et. al. (1978) is extended to include soft variables and used to illustrate the model construction methodology of section 3. Firstly a causal loop diagram is produced in line with the existing model (figure 1) and the additional hypotheses (section 2.2), and is given in figure 3.

The population variables are indicated as stocks, as there is no intention to remodel these. Solid connectors refer to the causal links of the original model, with the dashed connecters coming from the new hypotheses. This is an initial causal loop diagram pending the assessment of potential soft variables.

Feedback loops *R2* and *R3* are due to the effects of the media interest and rioter enthusiasm on recruitment. *R4* is due the effect of the media in creating initial sympathy for the riot. *R5* is the effect of enthusiasm on riot retention resulting from the numbers of rioters, whereas *B6* is the opposing force on retention resulting in the drop of enthusiasm due to arrests. *B3* is the balancing loop due to the loss of rioters due to arrests. *B5* is the effect of reducing enthusiasm through arrests in riot recruitment.

4.1 Identification of Soft Variables

The following variables are considered as potential soft variables: *Media Interest*, *sympathy in population towards rioters*, *Rioter Enthusiasm* and *events per day*. In addition *strength of cause* is also considered as a “soft constant”. Other model elements are population numbers related entities; all easily countable and with clear measures.

Media Interest

General Description. The media in the 1960s included newspapers and broadcasting. The media expresses its interest through its coverage of an event. If something is reported, the media is interested, and that reporting will have an effect on others.

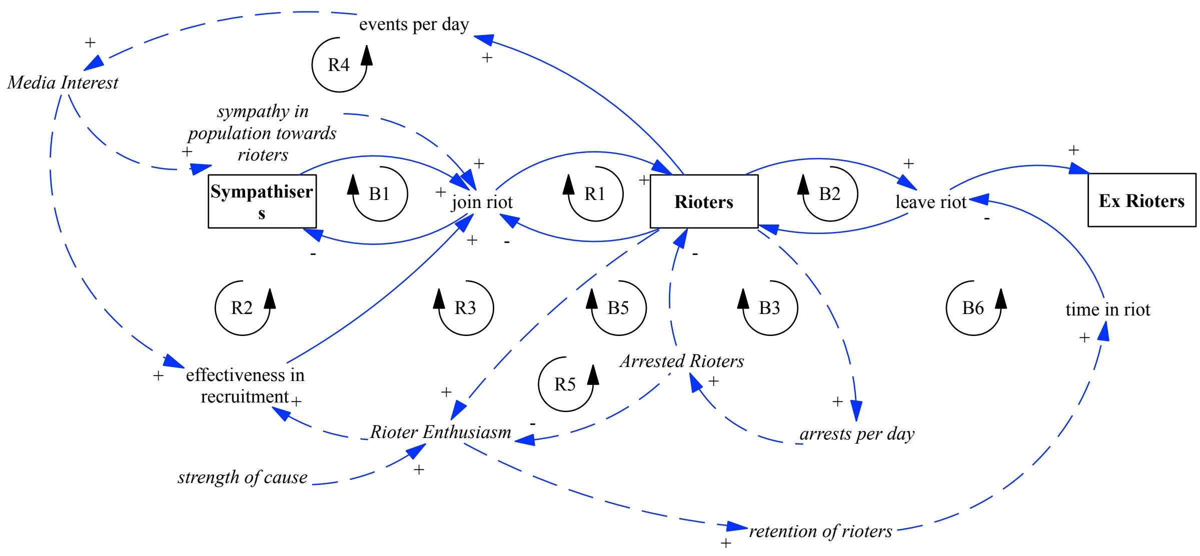

Figure 3: Initial CLD of the Extended Riot Growth Model

Aggregation. The media has different styles of reporting, in particular whether they are sympathetic or not to the event. As the only effect of media interest in this model is on a population of potential sympathisers for the riot and the rioters themselves it is deemed that any reporting would only positively affect sympathy and recruitment. The riots progressed so fast it was simply the news that they were happening that prompted people to action, not a more time-consuming weighing up the pros and cons. Thus a single variable should suffice.

Ordering. Newspapers and broadcasters give more coverage to some news items than others. A function of a newsroom each day is to prioritise news. Thus a clear ordering is implied.

Potential Measures. For newspapers these include: the number of words in the newspaper article; its position in the newspaper, e.g. main page, page 2 etc; its appearance in the opinion column. For broadcast media there could be: the duration of a broadcast report; its position in the running order; the number of reporters on the ground.

Thus *Media Interest* is a candidate for a soft variable.

Rioter Enthusiasm

General Description. Enthusiasm is a quality of an individual person, a rioter in this case. It describes an internal dedication to the riot at hand whose results will be seen by their desire to remain in the riot, recruit others and participate in protest. The latter is quite visible and could provide a meaning that others can agree with.

Aggregation. Enthusiasm can also be applied to the group of rioters. With individual enthusiasm dependent on external events such as riot size, strength of cause and arrests, there would not be a wide variation between rioters at a given point in time. Thus an average value of individual enthusiasm should suffice for group enthusiasm, giving the latter a realistic real world concept.

Ordering. It is common to describe some people as more enthusiastic than others, thus an ordered variable is reasonable.

Potential Measures. Potential measures of enthusiasm could be the volume of chanting by the crowd, verbal engagement with onlookers, people recruited per person, skirmishes with police. Generally individual enthusiasm for a cause can be measured by an amount of time in a given period spent on that cause, in this case time spent in the riot. Two of the measures mentioned, time in riot and people recruited per person, are effects of enthusiasm in the model.

Thus *Enthusiasm* is a candidate for a soft variable.

Sympathy

General Description. Sympathy is a quality of an individual person, a non-rioter in this case. It describes a positive engagement with any news reports about the cause coming from an internal alignment with the cause. It may result in positive verbal comments to others and an openness to join the cause. The former could provide a reportable meaning that others could agree.

Aggregation. Sympathy can also be applied to the *group* of non-rioters. However unlike enthusiasm, sympathy will have much wider variations over individuals at any one time; riots, and their causes, can invoke strong opinions for and against. Thus it is decided to disaggregate *Sympathisers*, into 2 stocks: one who are strongly sympathetic, and the remainder who remain to be persuaded. Although both could have a variety of thresholds of persuasions (Granovetter, 1978), to a first approximation each can be taken as a rough average of two widely diverging degrees of sympathy to the riot.

Thus the soft concept of *sympathy* is modelled by two population stocks, rather than a soft variable.

Strength of Cause

General Description. This is the strength of feeling in a person. It could be understood in terms of group membership, willingness to sacrifice time and money for the cause. There does not need to be a riot occurring for this to be understood.

Aggregation. If a single issue dominates the cause then a single variable is sufficient. The trigger for the riots was the deep feeling of racial bias in living conditions. Although there were many complex issues, this one issue summarised them all.

Ordering. Clearly some people will give more time and money to a cause than others.

Potential Measures. As mentioned, time and money spent on the cause, perhaps as a proportion of available time and money.

Thus *Strength of Cause* is a soft variable. As the riots progressed over short periods of time, a few days, it will be sufficient for it to be a constant in this model

Events per day

General Description. These were the events caused by rioters that were deemed illegal behaviour, largely damage, or attacks on other people. As events these are potentially countable and are only “semi-soft” because of their variety, including their seriousness.

Potential Measures. For each riot there are excellent measures of reported incidents to the police, medical and other authorities. Thus it will be sufficient to count these events, relying on the person who initiated the call to decide that they were serious.

As such *events per day* will be treated as a non-soft, countable variable; a population flow.

Now the potential soft variables have been identified, the causal loop diagram, figure 3 can be revised, figure 4. The two clear soft variables, *Media Interest* and *Rioter Enthusiasm*, are identified by stock notation. *Sympathisers* has been split into two stocks, with *Potential Sympathisers* earlier in the chain. The different effects of the media on creating sympathy *R4*, and enhancing recruitment to the riot *R2*, can now be seen more clearly.

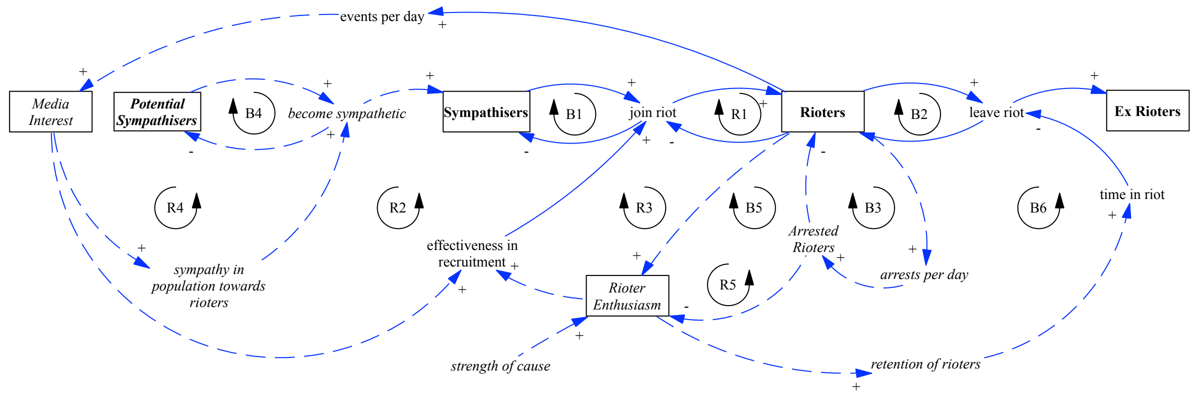

Figure 4: Revised CLD of Extended Riot Growth Model

4.2 Construction of Modular Structure

To assist the model construction, the causal loop diagram, figure 4, is re-expressed in modular form. The population model becomes a single module, named *Population with Riot,* which will contain all five population stocks. Loops, *R1*, *B1*, *B2*, *B3* and *B4* are internal to this module. The two soft variables become the other two modules, named *Enthusiasm of Rioters*, and *Media Interest*, figure 5 (overleaf).

There are three inputs to the population module, created by:

i. The effect of *media interest* on sympathy for the riot *R4*;

ii. The combined effect of *media interest R2,* and *rioter enthusiasm R3 & B5,* on recruitment;

iii. The effect of *rioter enthusiasm* on rioter retention *R5 & B6.*

The three effects, which are the interface between the soft variable models and the population module, are themselves made modules.

The population module also has two outputs:

i. The negative effect of arrests on *rioter enthusiasm B5 & B6;*

ii. The positive effect of riot size on *rioter enthusiasm R3 & R5*, combined with the exogenous strength of the cause. Additionally there is a positive effect of riot size on *media interest R2 & R4.* This output is used twice.

The three effects of the population are passed through interfaces to the two soft variables, again made modules to model those effects.

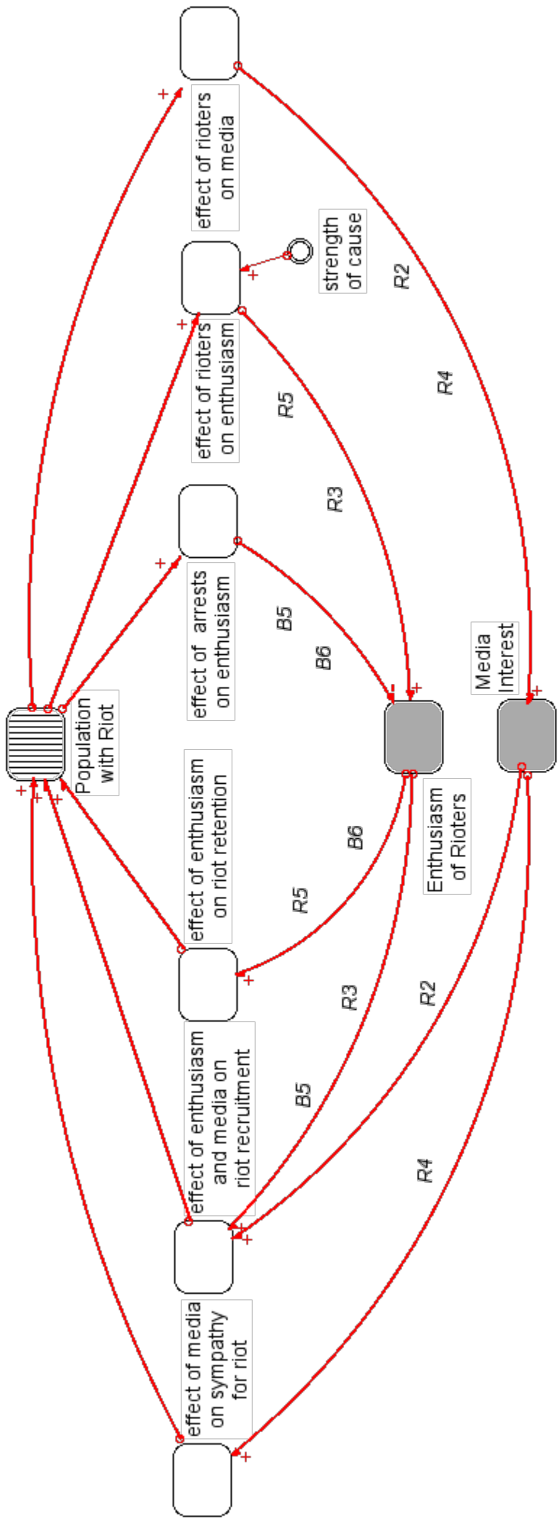

Figure 5: Modular CLD of Extended Riot Growth Model

4.3 Construction of Sub-Models

4.3.1 Population Sub-Model

The main population sub-model is straightforward, figure 6. The two outputs, *Rioters* and *Arrested Rioters*, are indicated by double outline symbols. The three inputs are given in bold, and contain the name of the connecting sub module "*effect of …*". The three inputs are the *fraction of the population influenced*, the *probability of successful influence* (of the sympathisers) and *time in riot*. The three are exogenous parameters as far as the sub-model is concerned, but endogenous in the overall model. If these parameters are set exogenously, e.g. by constants or time series, the sub-model can be tested independently. There are two internal parameters, *Potential Number of Contacts* and *Arrest rate*; and the five initial values.

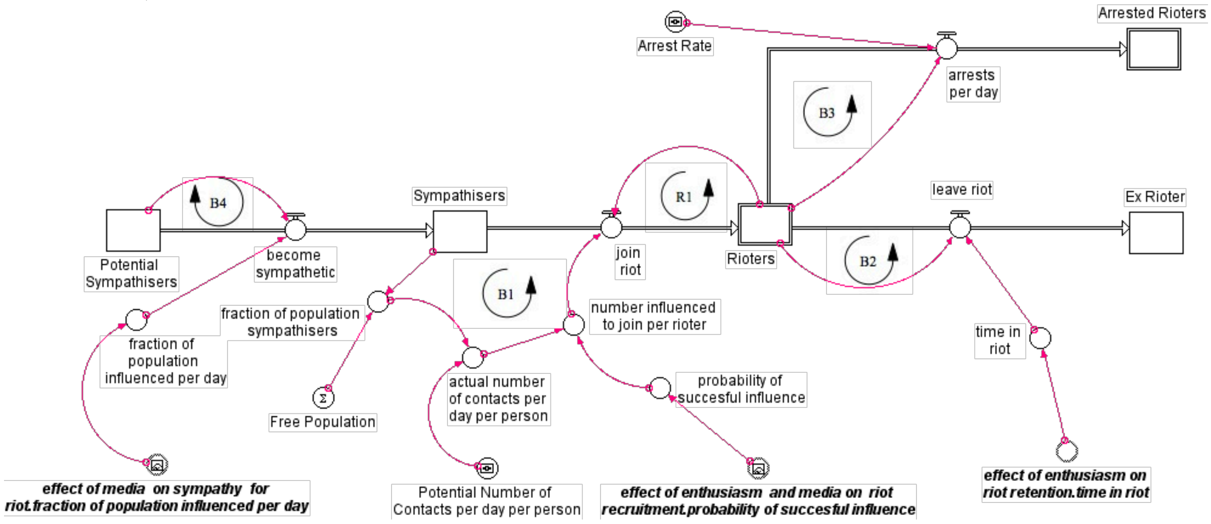

Figure 6: Population Sub Model

4.3.2 Media Interest Sub-Model

As a soft variable the construction of *Media Interest* will require a consideration of scale, units, nature, inputs and output.

Scale

There is a clear minimum value *Media Interest*, corresponding to no reporting by the media. Since newspapers and broadcasters have a limited capacity then *Media Interest* will be given a maximum value.

Units

If units are the number of words used as a fraction of the total in a paper, then the minimum value is 0%, and the maximum is 100%. A similar argument follows if the units are the percentage of the broadcast time.

Nature

Because reports can be hourly, or daily, it is possible to think of *Media Interest* as a flow. However *Media Interest* is not just modelling the physical reporting of the events, but the impact those events have on the editors. As such reports of events will appear in subsequent news reports, often with diminishing importance, unless there has been new events of the same nature. Thus there is a group memory effect of the editorial team, and thus the variable is best modelled as a stock.

Additionally *Media Interest* is also understood by its effect on a population. That population will also have a memory of news items for some period. Thus a stock is preferred.

In order to achieve a limited scale on a stock, a goal-seeking archetype is used, figure 7. The balancing loop *B1m* controls the growth of the stock, limiting growth to the target, *maximum media interest*. The loop *B2m* allows the *Media Interest* to decline if there are no further events to stimulate interest, modelling the decline in reporting over time when a news item becomes "old news". The generic pattern will be referred to as a Soft Variable on a Limited Scale (SVLS). Levine (2000) uses a similar goal-seeking construct for soft variables.

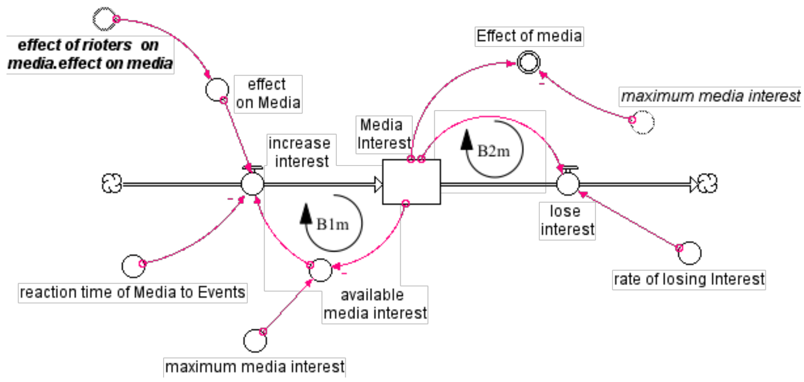

Figure 7: Media Interest Sub-Model

Inputs

Media Interest is only influenced by the effect of the rioters. This causes increased interest from the media, and is connected to the inflow (shown in bold, figure 7).

Outputs

The output is related to the stock value. In order to hide both values and units the stock value is divided by its maximum possible value. The output, indicated with a double outline, figure 7, is now limited to a scale 0 to 1, regardless of the value of the stock. Thus any potential measure of the soft variable is private, and does not affect the use of the variable in the model.

4.3.3 Rioter Enthusiasm Sub-Model

This follows in a similar fashion to *Media Interest*.

Scale

The minimum value of *Rioter Enthusiasm* occurs when all individuals have no enthusiasm, as expressed by them not recruiting anyone from a sympathetic population, and leaving the riot in the shortest possible time. A maximum value follows from the limited capacity of a person to engage in any activity. If a person reaches that capacity then there would be no reason to envisage more enthusiasm as that extra enthusiasm would have no effect on any dynamical element.

Units

Once a maximum has been set then a percentage of the maximum would be a natural unit. However other units could be constructed, such as the fraction of rioters involved in skirmishes with the authorities.

Nature

Enthusiasm for a cause can be generated internally as well as from external stimuli. As the cause in this case is a serious breach of the peace by a group, it will be assumed that unless there is an external cause then that enthusiasm to participate in a riot will fade. Thus a stock is the most natural element, with any internal reinforcement mechanisms being covered by the memory effect of a stock

Thus *Rioter Enthusiasm* is constructed in a similar fashion to *Media Interest*, using the soft variable on a limited scale model, figure 8.

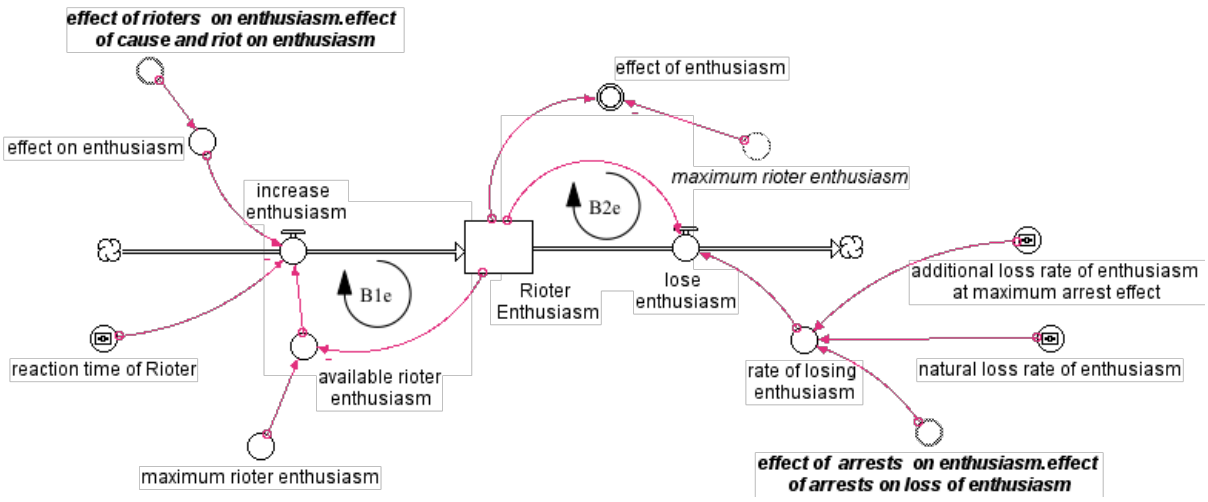

Figure 8: Rioter Enthusiasm Sub-Model

Inputs

Rioter Enthusiasm has two inputs: the positive effect of the number of rioters; the negative effect of the number of arrests. The former is connected to the inflow, and the latter to the outflow affecting the proportional rate of loss from the stock (indicated in bold). A parameter to represent the natural loss of enthusiasm in the absence of any arrests is required. The effect of arrests on enthusiasm is over and above this natural rate, controlled by the parameter *additional loss rate of enthusiasm at maximum arrest effect.*

Outputs

The output is related to the stock *Rioter Enthusiasm*. Like *Media Interest* division by the maximum value achieves the privacy of the units and scale of enthusiasm, constraining the scale to 0 to 1.

Thus a case has been made for both soft variables to have dimensionless outputs on a scale 0 to 1. Coyle (2000) described such a scale as tempting but questioned the meaning of a value of 0.5. In the models above such a value would mean that the variables have 50% of the effect that they would have had at their maximum value. However it does not require that any measure of the variable would be at 50% of its scale as the measure is related to the internal stock value, which may not have a linear relationship, nor even be a ratio scale (Levine, 2000). For example if the output of media interest is 50% then it means that the effect of the media is half what it could potentially be. It does not mean that half the newspaper is filled with news of the riot.

As the model is constructed the input also places an interpretation on the meaning of the value 50%. Because of the loop *B1m* it would be harder to raise the value from 50%, than to reduce the value. Put another way, it takes less effort to raise the soft variable value to the halfway point than to take it from halfway to saturation. It has been assumed that as the media gets close to saturation there is a greater reluctance to increase coverage. Alternative models could be constructed if a linear relationship were preferred.

These approaches to the interpretation of the soft variable value raise important issues for model calibration. The parameters connected with the soft variable should be determined by its response to the causes, and the effect of the variable in the model, not necessarily by any potential measures. In this sense the soft variable is referentially transparent, that is, in the model it is understood in terms of its inputs and outputs alone. For example in the case of *Rioter Enthusiasm* then in two different runs, with identical inputs, the output should be the same for the same values of its two internal parameters, *reaction time to Rioter hostility* and the initial value of the variable. The latter is scaled in a similar way to the output to preserve privacy. The sub-model can then be used without any knowledge of its internal workings, apart from the parameter values.

4.4 Use of Soft Variables

Having established that the effect of a soft variable can be distinguished from its measures, then modules to control the effect of each soft variable on the population module are introduced.

Effect of Soft Variables on Population

Media interest has an effect on the potential sympathetic population; rioter enthusiasm affects rioter retention (figure 5). There is a third effect of both soft variables combined on recruitment to the riot. The output of both soft variables is on a 0 to 1 scale, thus converting the output into an effect is relatively straightforward and the effects can be combined using the algebra of table 1.

The media effect on the sympathy of the potential rioters is by parameter multiplication, a converter to convert media on its normalised scale to sympathy, figure 9. This in turn links to the fraction of the population influenced per day, which is handled by a graphical converter to allow for potential non-linearities in the effect, and to deal with the change of units from [unitless] to [day]^(-1).

For the effect of the enthusiasm, E, on the average time a rioter spends in the riot, t_{riot}, a maximum, $t_{\max}$, and minimum time, $t_{\min}$, is used, $t_{riot} = t_{\min} + (t_{\max} - t_{\min})E$, figure 10. The 0 to 1 scale of E ensures the bounds are not exceeded. The dimensions balance, as the effect of the soft variable is unitless.

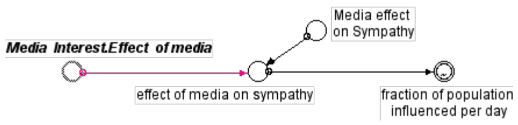

Figure 9: Media Effect on Sympathy

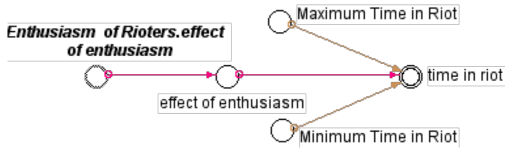

Figure 10: Enthusiasm Effect on Riot Retention

The effect of both soft variables on the probability of a successful influence by a rioter P_{succ} requires some thought as to how the two variables combine. The riots in question took place in the 1960s before the advent of social media, and occurred over a very short period of time, thus direct recruitment through media exposure has been ruled out of this model. Instead media exposure enhances the likelihood of successful recruitment. It is assumed that media exposure would make a sympathiser more likely to accept, and rioter enthusiasm would enhance a rioter's ability to persuade.

Thus it is conceivable that the media interest M would have been sufficient to cause rioters to recruit people to the riot, even if their enthusiasm was zero. This would be the situation where a rioter merely informs the sympathisers of the fact that the riot is taking place. That is, there is a demand from people to join the riot, provided someone tells them the place. Likewise a riot is possible without any media exposure to enhance the likelihood of acceptance, as the persuasion of the rioter would be sufficient. Thus OR logic, the lenient combination, is deemed the most suitable.

Let the effect of the media be scaled by a parameter, *media effect on recruitment* $e_M \le 1$. Likewise scale the effect of the enthusiasm, *enthusiasm effect on recruitment* $e_E \le 1$. Then the probability of successful influence is $P_{succ} = e_M M + e_E E - e_M M e_E E$, figure 11. The limited scale of the two soft variables, and the two associated parameters, ensures the scale of the probability lies on 0 to 1. The two parameters allow the relative effects of the two soft variables on recruitment to be adjusted.

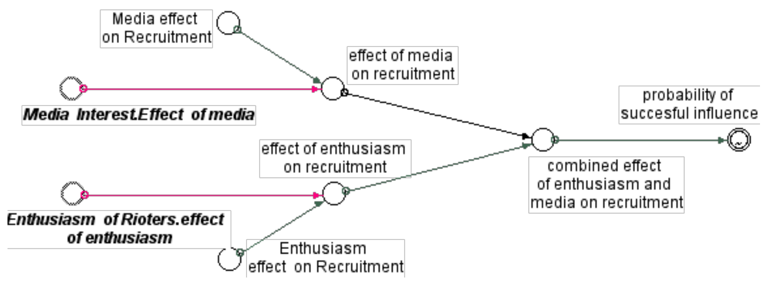

Figure 11: Effect of Media and Enthusiasm on Rioter Recruitment

There are other potential models of how these two variables could be combined.

Effect of Population on Soft Variables

The population has two effects on rioter enthusiasm; positively through the numbers in the riot, and negatively through the arrests. Enthusiasm has two inputs to accommodate this. The positive effect on enthusiasm is also affected by the exogenous strength of the rioters' cause. The population also has an effect on the media interest. There are three separate outputs from the population module for these three effects.

Three separate modules provide the linkage between the population module and the three effects. Unlike the soft variables the population variables are not hidden, nor are they on a limited scale. Thus each of the effect modules will need to translate the population outputs into suitable scales. This is achieved using a Holling function of type II, $f = x/(x_{half} + x)$ (Holling, 1959; Freedman, 1980). The parameter x_{half} is the value of the input x that produces an output of 0.5, unity being the maximum output. The curve is monotonic of decreasing positive gradient. It corresponds to the logit term (Sterman, 2000). x_{half} governs the strength of the effect of the input on the output.

The effect of arrests on rioter enthusiasm follows the Holling construct, figure 12. Increasing arrests gives an increasing output, whose effect on Enthusiasm is to subtract, figure 8.

The effect of the riot size on Media interest is via *events per day*, which is a flow leading to the cumulative reported events, figure 13. A parameter, *events per rioter per day*, controls the impact of the rioters on such events. *Events per day* is passed through a Holling type II function to achieve a normalised scale output.

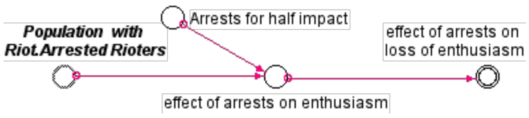

Figure 12: Effect of Arrests on Enthusiasm

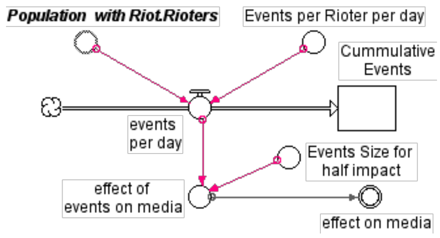

Figure 13: Effect of Rioters on Events

The final module contains the combined effects of the size of the riot e_R and the strength of the cause that underlies the riot C. C and e_R do not have a symmetric effect on rioter enthusiasm. If there is no riot $e_R = 0$, a cause worth fighting for $C > 0$ is sufficient to produce enthusiasm: $e_{RC}(C, e_R) = e_{RC}(C, 0) > 0$. However if there is no cause $C = 0$, then a riot will not of itself produce enthusiasm in the rioters $e_{RC}(0, e_R) = 0$. It might encourage some to join in with looting, but to engage with the authorities there would need to be some grievance, or cause to fight for. The combination needs a cause, and an occurring riot enhanced by the cause.

The strength of the cause, and the effect of riot size are placed on a scale 0 to 1, the latter again achieved with a Holling function. The riot enhanced by the cause is given by OR logic, table 1, $e_{RorC} = e_R + C - e_R C$. The final output is achieved with AND logic between this and the strength of cause: $e_{RC} = e_{RorC} \times C = (e_R + C - e_R C)C$, given by *effect of cause and riot on enthusiasm* in figure 14. This satisfies $e_{RC}(C, 0) > 0$ and $e_{RC}(0, e_R) = 0$. However large the riot, even if $e_R = 1$, its maximum value, then e_{RC} cannot be unity, unless the strength of the cause is unity. Thus however large a riot, it cannot have the maximum effect on rioter enthusiasm unless the strength of the cause is total.

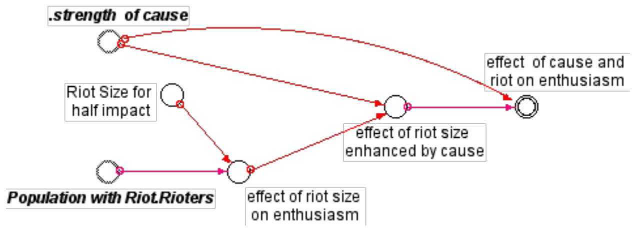

Figure 14: Effect of Rioters and Strength of Cause on Enthusiasm

The full model code is given in the appendix, with Stella 10 files in the additional material.

5 Model Testing

5.1 Soft Variable on a Limited Scale Sub-Model

A generic single stock model was used for both media interest and the enthusiasm of rioters, referred to as a soft variable on a limited scale model (SVLS). The model has a goal seek balancing loop to control the growth and a draining process on the outflow to allow for decline in the absence of any input. An alternative model for soft variables is the standard stock adjustment (SA) process, where the input is through the goal of the process. Of course this latter process cannot handle a limited scale, unless the limits are achieved within the calibration window of the model.

To test the SVLS sub-model, its response to standard inputs is compared with the stock adjustment process, figure 15. Two such results are shown in figure 16. For a low input value, that keeps output significantly less than unity, the SVLS can follow stock adjustment for a suitable choice of parameters (left hand graph). However when the input value is increased, the output of the SVLS is compressed to keep it under unity, the intended result (right hand graph). By contrast the stock adjustment output has exceeded unity. Thus the SVLS is an improved version of stock adjustment if the scale is to be limited, producing similar responses and delays.

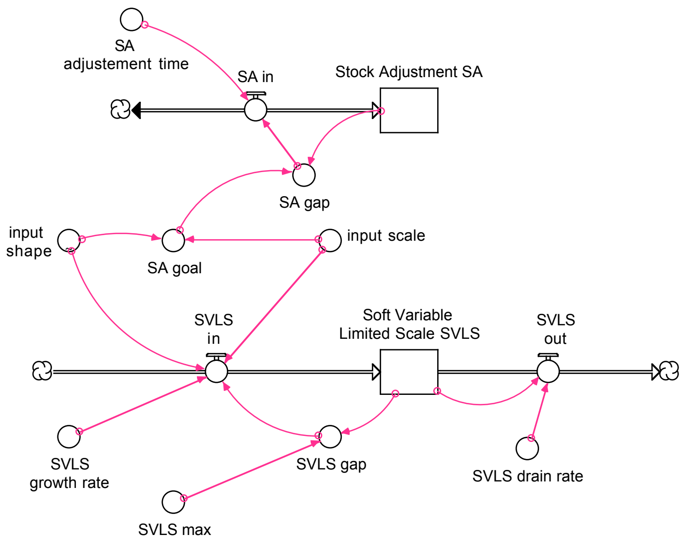

Figure 15: Stock Adjustment Process & Soft Variable on Limited Scale Model

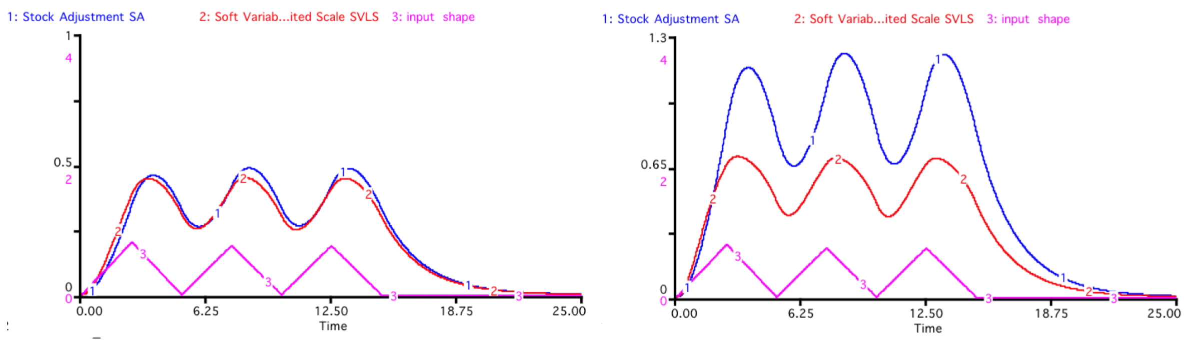

Figure16: Comparison of Stock Adjustment with Soft Variable on Limited Scale Model

5.2 Calibration & Historical Fit

The starting point for calibration is the historical fit to the reported events of the Los Angeles 1965 riot for the original model, as given in Burbeck et. al. (1978). This fit was reproduced with the loops through *Media Interest* and *Enthusiasm of Rioters* switched off, figure 17. Additionally the attempt was made to ensure around 30,000 people participated in the riot (Burbeck et. al., 1978). The effects due to the new hypotheses were introduced gradually until the desired effect of the soft variables was reproduced. At each stage the historical fit of reported events per day[1], figure 17, was the benchmark for re-calibration.

1. The arrest rate was calibrated to give the total arrests after 5 days of about 4000 people (Martin Luther King Jr., 2014).

2. A realistic graph for media interest was obtained; starting at zero it rises rapidly, but delayed following the onset of the riot (figure 18). It peaks after the peak of the riot, and does not return to zero when the riot has ceased, as there was still significant media coverage.

3. A realistic graph for rioter enthusiasm was produced. This was assumed to start high, rise and peak before the peak of the riot due to the freshness of the original enthusiastic rioters, and then fall slowly due to the effect of the arrests, and the dwindling riot numbers (figure 18). Care was taken to ensure the strength of the cause and the riot numbers both contributed to the rise in enthusiasm. The fixed strength of cause has the effect of reproducing the contour of the effect of the rise in rioters, but squashing the combined response into a narrower range, thus making the change of enthusiasm respond to riot size in a less than linear fashion (figure 19). This follows from the combination $e_{RC} = (e_R + C - e_R C)C$ discussed earlier.

4. The effect of the media on producing sympathisers was slowly calibrated so that *Potential Sympathisers* initially matched the sympathisers, figure 20. *Sympathisers* starts by rising before the effects of riot recruitment deplete its numbers. This left about 20,000 people who did not participate in the riots, a much larger figure than

[1] The reported events per hour were compiled by Burbeck et. al., (1978), and appear in that paper. For this paper the units have been changed to events per day, figure 17.

that obtained by Burbeck et el (1978) who had criticised their own data fitting for depleting too much of the sympathetic population. Thus the introduction of the *Media Interest* soft variable, and the concept of sympathy, has improved the Burbecks's original model.

5. The effect of enthusiasm on the time in the riot was calibrated to give a small variation, where the average of the minimum and maximum values was similar to the *time in riot* obtained by Burbeck et. al. (1978), figure 21. This enabled a moderate effect of falling enthusiasm on rioter retention. There would be scope at this point for alternative calibrations in order to produce the asymmetry seen in other documented rioter numbers (Burbeck et. al., 1978).

6. Finally the effect of enthusiasm and media on rioter recruitment was introduced. The effect of the media was adjusted to exceed the effect of enthusiasm by around the end of the second day when the media exposure had become established, figure 22. The final historical fit for events per day, figure 17 was reproduced again. Parameter values are given in table 2.

Module	Parameter	Values
Global	strength of cause	0.5
effect of rioters on enthusiasm	Riot Size for half impact	400
effect of arrests on enthusiasm.	Arrests for half impact	600
effect of rioters on media.	Events Size for half impact	30
	Events per Rioter per day	0.01
effect of enthusiasm on riot retention	Minimum Time in Riot	0.1
	Maximum Time in Riot	0.42
effect of enthusiasm and media on riot recruitment	Media effect on Recruitment	1.0
	Enthusiasm effect on Recruitment	0.82
effect of media on sympathy for riot	Media effect on Sympathy	1.0
Media Interest	reaction time of Media to Events	0.2
	rate of losing Interest	0.7
	Initial Media effect	0.0
Enthusiasm of Rioters	reaction time of Rioter	0.3
	natural loss rate of enthusiasm	0.4
	Initial effect of Enthusiasm	0.5
Population with Riot	Potential Number of Contacts per day per person	18.9
	Arrest Rate	0.49
	Total Population	50,000
	Initial Sympathisers	25,000
	Initial Rioters	90

Table 2: Parameter Values for Historical Data Fit, Los Angeles Riot 1965

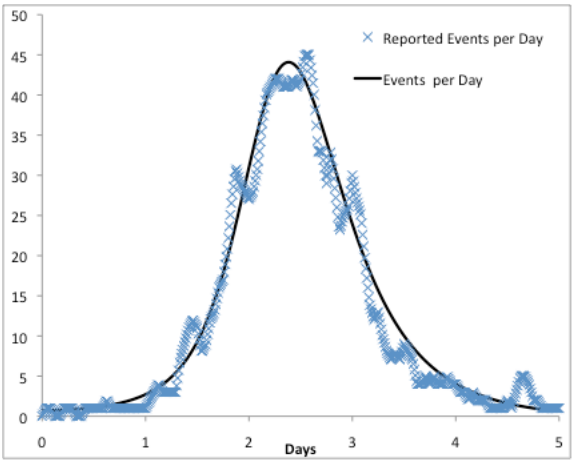

Figure 17: Reported Events Compared with Model

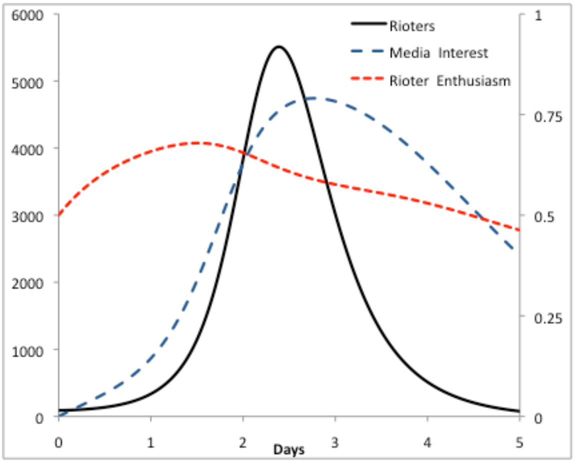

Figure 18: Rioters, Media and Enthusiasm

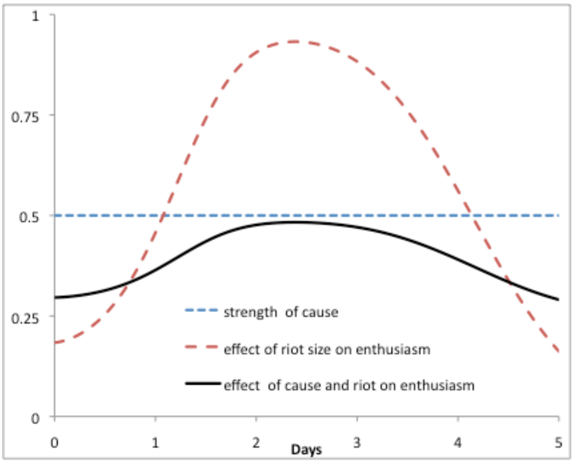

Figure 19: Effect of Riots on Enthusiasm

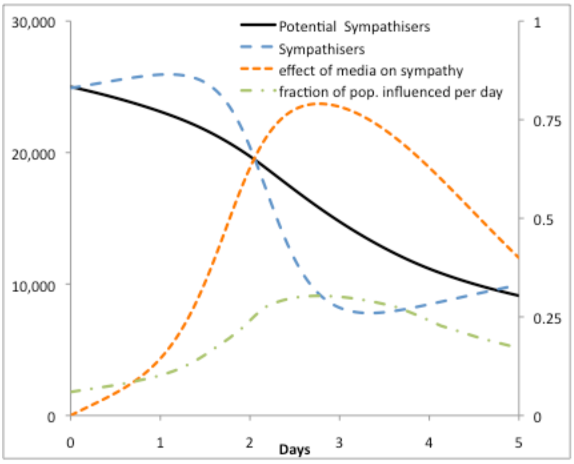

Figure 20: Effect of Media on Sympathy

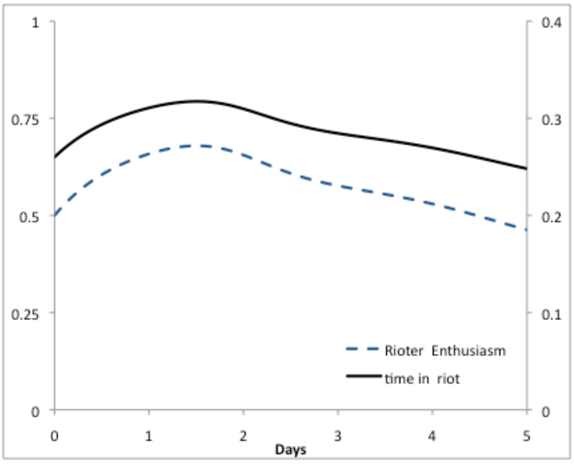

Figure 21: Effect of Enthusiasm on Time in Riot

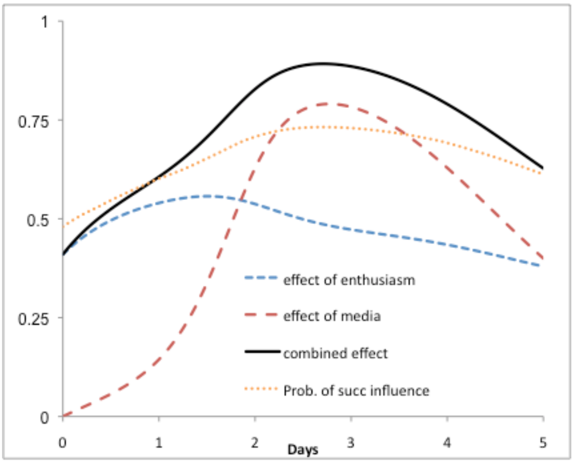

Figure 22: Combined Effects on Recruitment

6 Discussion

6.1 Model Improvements

The inclusion of soft variables into the Burbeck et. al. (1978) model for the dynamics of riot growth has improved the model in two ways. Firstly there are improved parameter values. The initial number of rioters has increased from 10, which Burbeck et. al. admitted was too low, to 90. This follows from the effects of rioter enthusiasm and media interest on riot recruitment and retention. It is quite conceivable that the initial number may improve even further with more refined data fitting. Additionally, the total number of sympathisers has increased dramatically from the Burbeck et. al. value, which was only just above the total number of rioters. With the inclusion of the soft variables a realistic historical fit is obtained with about half the initial sympathisers never joining the riot, closer to the recorded facts. This followed from the effects of media interest on the potentially sympathetic population.

The second improvement in the model is that it now tells a better story. Burbeck et. al. admitted that the probability of rioter recruitment and retention were unlikely to be constant, and that their enthusiasm played a part in the dynamics. Likewise the sympathy of the potential population was believed to increase with media exposure. Soft variables have allowed the model to reproduce that story.

6.2 Methodological Improvements

The methodology presented has made at least five potential improvements over existing ones. Firstly, the use of data hiding in the methodology has helped to separate out the <u>measurement</u> of soft variables from their <u>use</u>. Hiding the soft variable units has enabled the effects of the variables to be handled without being confused by their measurement schemes. This approach now encourages the modeller to think about the relationship of measures of the soft variable independently of its use.

Consider an analogy from physics. Solar physicists require measures of key variables of the sun such as temperature, pressure etc. These variables interact with each other dynamically as the solar composition changes. The variables cannot be measured directly but indirect measures can be obtained by the radiation that is produced. However the measures of the radiation are not used in the dynamical computation of the sun, they are only an indication of what the underlying variables are. Likewise measures of soft variables are not directly used in the construction and calibration of a system dynamics model, but are only an indication of how those variables behave.

As a specific example consider the case where a measure of *Media Interest* has been obtained in terms of length of radio broadcasts. A general measure of the media can be modelled as a separate output from the *Media Interest* module, figure 23a. The *Media Interest* module is then connected to a module dedicated to the radio broadcast measure of the media, figure 23b. The radio broadcast module, receives the hidden value of *Media Interest* as input and then models the relationship between it and the length of radio broadcasts by whatever means, functional or look up table, possibly non-linear. Finally length of radio broadcast is output, figure 23c. This preserves the privacy of the *Media Interest* soft variable, whilst making the measure available. The calibration of the model will not directly depend on the <u>values</u> of this measure, although indirectly, together with its own model of its relationship to the soft variable, it could tighten up a calibration.

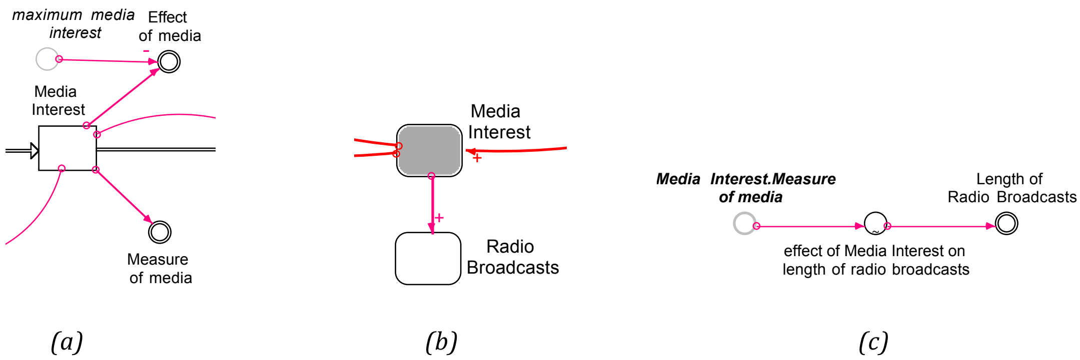

Figure 23: Measurement of a Soft Variable. (a) Media Module with Separate Outputs for Measure & Effect. (b) Modular View of Media Interest & Radio Broadcast. (c) Model of Radio Broadcast Measure of Media

The second methodological improvement is that data hiding has enabled different soft variables with a limited scale to be combined on a common scale, normalised to 0 to 1. The normalisation of the output indicates the maximum effect the soft variable has on other system elements, not influenced by its measure. Thus cognitive algebra and probability laws can be employed to create a more systematic approach to variable combination. Although in this current study soft variables have been restricted to a limited scale, the maximum of the scale is not limited as it is hidden.

Thirdly, the use of referential transparency has allowed the sub-modules encapsulating the soft variables to be tested prior to their inclusion in the model. Likewise the modules for the causes and effects of the variables can be independently tested in their different combinations. The inner details of the models are not required to be seen for the sub-modules to be used; a knowledge of their parameters is sufficient. Thus in the model the soft variable is understood by its use and behaviour, not its internal construction.

Fourthly, following on from referential transparency, model calibration has been made more systematic. The effects of the population on the soft variables can be introduced gradually before their feedback on the population is introduced. With slow calibration from the prior historical fit without soft variables, the model user only needs to concentrate on a small number of adjustable parameters at any one time.

Fifthly the staged procedure for the identification and construction of soft variables brings clarity to the modelling process for both other modellers and clients involved in the process. Undoubtedly the procedure presented here is not unique in its order or number of stages. However such an agreed procedure would bring more confidence to the model building and make it easier to deal with ambiguities and errors in the modelling process

7 Conclusion

A procedure for the identification, construction and use of soft variables has been proposed. A modular structure was used to encapsulate the soft variables enabling their values and units to be hidden, thus separating out the effect of the soft variable from its measure. The modular system enforces referential transparency that each soft variable is understood by the relationship between its inputs, and outputs alone, for given parameter values. The method was applied to an existing SIR type model of the progress of a riot which employed only population, i.e. non-soft, variables. The method also enabled the population to be encapsulated in a module with a state composed of a number of stocks, i.e. multi-dimensional. It is felt the method shows promise as it produced an improved model, with a sharper understanding of the identification, construction and use of soft variables.

Although beyond the scope of the current paper, there is much that could be done with sensitivity analysis to produce alterative historical fits and stories. In particular it would be interesting to explore a greater variation on the *probability of successful influence*, and the *time in riot*, which are quite conservative in the above results. Although a systematic approach to calibration was tried, there would be much scope for improvement here. Additionally the model could be applied to other riots, especially those where there is a degree of asymmetry in their rise and fall (Burbeck et. al., 1978). Nevertheless it is felt the current analysis is sufficient to create an interest in this type of methodology. As such it would be interesting to apply the method to other scenarios.

It is not claimed that this methodology is complete, explored fully, or better than existing attempts to include soft variables. Instead it is hoped that the paper will inspire further research in the area.

Acknowledgements

Soft variables have been used extensively in student projects on the undergraduate system dynamics course at the University of South Wales for a number of years. Particular thanks go to former students Dr Alex Berriman, Dr John Mehers, Amira Irshad and Catherine Evans for the helpful contributions they made to the methodology given in this paper. Thanks are also due to Professor Paul Roach, of the same university, for inspiring discussions on the use of cognitive algebra.

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Appendix – Model Code & Units

Hours_per_day = 24
Units: Hours per Day
Reported_events = GRAPH(time_in_hours)
(0.00, 0.00), (1.00, 1.00), (2.00, 1.00), (3.00, 0.00), (4.00, 0.00), (5.00, 1.00), (6.00, 1.00), (7.00, 1.00), (8.00, 0.00), (9.00, 0.00), (10.0, 1.00), (11.0, 1.00), (12.0, 1.00), (13.0, 1.00), (14.0, 1.00), (15.0, 2.00), (16.0, 1.00), (17.0, 1.00), (18.0, 1.00), (19.0, 1.00), (20.0, 1.00), (21.0, 1.00), (22.0, 1.00), (23.0, 1.00), (24.0, 1.00), (25.0, 2.00), (26.0, 3.00), (27.0, 4.00), (28.0, 3.00), (29.0, 3.00), (30.0, 3.00), (31.0, 3.00), (32.0, 7.00), (33.0, 9.00), (34.0, 11.0), (35.0, 12.0), (36.0, 11.0), (37.0, 8.00), (38.0, 9.00), (39.0, 12.0), (40.0, 13.0), (41.0, 16.0), (42.0, 17.0), (43.0, 21.0), (44.0, 27.0), (45.0, 31.0), (46.0, 29.0), (47.0, 28.0), (48.0, 27.0), (49.0, 28.0), (50.0, 31.0), (51.0, 36.0), (52.0, 40.0), (53.0, 41.0), (54.0, 42.0), (55.0, 42.0), (56.0, 41.0), (57.0, 41.0), (58.0, 42.0), (59.0, 41.0), (60.0, 42.0), (61.0, 45.0), (62.0, 45.0), (63.0, 41.0), (64.0, 33.0), (65.0, 33.0), (66.0, 29.0), (67.0, 33.0), (68.0, 28.0), (69.0, 23.0), (70.0, 25.0), (71.0, 26.0), (72.0, 30.0), (73.0, 27.0), (74.0, 25.0), (75.0, 19.0), (76.0, 14.0), (77.0, 12.0), (78.0, 13.0), (79.0, 10.0), (80.0,

8.00), (81.0, 7.00), (82.0, 8.00), (83.0, 7.00), (84.0, 9.00), (85.0, 8.00), (86.0, 7.00), (87.0, 4.00), (88.0, 4.00), (89.0, 5.00), (90.0, 4.00), (91.0, 5.00), (92.0, 4.00), (93.0, 4.00), (94.0, 5.00), (95.0, 4.00), (96.0, 4.00), (97.0, 3.00), (98.0, 3.00), (99.0, 2.00), (100, 3.00), (101, 2.00), (102, 2.00), (103, 2.00), (104, 1.00), (105, 1.00), (106, 1.00), (107, 1.00), (108, 2.00), (109, 1.00), (110, 3.00), (111, 5.00), (112, 5.00), (113, 4.00), (114, 2.00), (115, 2.00), (116, 1.00), (117, 1.00), (118, 1.00), (119, 1.00), (120, 1.00)
{ events per hour as given in Burbeck et al, 1978}
Units: Hours
strength__of_cause = 0.5
Units: Dimensionless
time_in_hours = time*Hours_per_day
Units: Hours

---

effect of enthusiasm on riot retention:
effect_of_enthusiasm = Enthusiasm__of_Rioters.effect_of_enthusiasm
Units: Dimensionless
Maximum_Time_in_Riot = 0.42
Units: Days
Minimum_Time_in_Riot = 0.1
Units: Days
time_in_riot = Minimum_Time_in_Riot+(Maximum_Time_in_Riot-Minimum_Time_in_Riot)*effect_of_enthusiasm
Units: Days

---

effect of enthusiasm and media on riot recruitment:
combined_effect_of_enthusiasm_and_media_on_recruitment = effect_of_enthusiasm_on_recruitment +effect_of_media__on_recruitment - effect_of_enthusiasm_on_recruitment*effect_of_media__on_recruitment
Units: Dimensionless
effect_of_enthusiasm_on_recruitment = Enthusiasm_effect__on_Recruitment*Enthusiasm__of_Rioters.effect_of_enthusiasm
Units: Dimensionless
effect_of_media__on_recruitment = Media_effect_on_Recruitment*Media_Interest.Effect_of_media
Units: Dimensionless
Enthusiasm_effect__on_Recruitment = 0.82
Units: Dimensionless
Media_effect_on_Recruitment = 1
Units: Dimensionless
probability_of_succesful_influence = GRAPH(combined_effect_of_enthusiasm_and_media_on_recruitment) (0.00, 0.308), (0.1, 0.35), (0.2, 0.389), (0.3, 0.432), (0.4, 0.474), (0.5, 0.53), (0.6, 0.598), (0.7, 0.65), (0.8, 0.697), (0.9, 0.735), (1, 0.803)
Units: Dimensionless

---

effect of media on sympathy for riot:
effect_of_media_on_sympathy = Media_effect__on_Sympathy*Media_Interest.Effect_of_media
Units: Per Day
fraction_of_population_influenced_per_day = GRAPH(effect_of_media_on_sympathy) (0.00, 0.0597), (0.1, 0.0903), (0.2, 0.116), (0.3, 0.144), (0.4, 0.171), (0.5, 0.197), (0.6, 0.227), (0.7, 0.274), (0.8, 0.306), (0.9, 0.334), (1.00, 0.369)
Units: Per Day
Media_effect__on_Sympathy = 1
Units: Per Day

---

effect of rioters on enthusiasm:
effect_of_riot_size_enhanced_by_cause = effect_of_riot_size_on_enthusiasm+.strength__of_cause-effect_of_riot_size_on_enthusiasm*.strength__of_cause
Units: Dimensionless
effect_of_riot_size_on_enthusiasm = Population__with_Riot.Rioters/(Riot_Size_for__half_impact+Population__with_Riot.Rioters)
{Holling type 2 function - impact is absolute not relative total population}
Units: Dimensionless
effect__of_cause_and_riot_on_enthusiasm = effect_of_riot_size_enhanced_by_cause*.strength__of_cause
{either cause, or protest size enhanced by cause - thus cause is sufficient to provide enthusiasm, but size with no cause is not}
Units: Dimensionless
Riot_Size_for__half_impact = 400
Units: People

---

effect of rioters on media:
Cummulative_Events(t) = Cummulative_Events(t - dt) + (events__per_day) * dt
INIT Cummulative_Events = 0
Units: Events
INFLOWS:
events__per_day = Events_per_Rioter_per_day*Population__with_Riot.Rioters
Units: Events/Days
effect_of__events_on_media = events__per_day/(events__per_day+Events_Size_for__half_impact)
Units: Dimensionless
effect_on_media = effect_of__events_on_media
Units: Dimensionless
Events_per_Rioter_per_day = 0.008
Units: Events per rioter per day

Events_Size_for__half_impact = 30
Units: Events/Days

---

effect of arrests on enthusiasm:
Arrests_for_half_impact = 600
Units: People
effect_of_arrests_on_enthusiasm =
Population__with_Riot.Arrested_Rioters/ (Population__with_Riot.Arrested_Rioters+Arrests_for_half_impact)
Units: Dimensionless
effect_of_arrests_on_loss_of_enthusiasm = effect_of_arrests_on_enthusiasm
Units: Dimensionless

Enthusiasm of Rioters:
Rioter__Enthusiasm(t) = Rioter__Enthusiasm(t - dt) + (increase__enthusiasm - lose__enthusiasm) * dt
INIT Rioter__Enthusiasm = Initial_effect_of_Enthusiasm*maximum_rioter_enthusiasm
Units: Enthusiasm
INFLOWS:
increase__enthusiasm = effect_on_enthusiasm*available_rioter__enthusiasm/reaction_time_of_Rioter
Units: Enthusiasm/Days
OUTFLOWS:
lose__enthusiasm = Rioter__Enthusiasm*rate_of_losing__enthusiasm
Units: Enthusiasm/Days
additional_loss_rate_of_enthusiasm_at_maximum_arrest_effect = 1
Units: Per Day
available_rioter__enthusiasm = (maximum_rioter_enthusiasm-Rioter__Enthusiasm)
Units: Enthusiasm
effect_of_enthusiasm = Rioter__Enthusiasm/maximum_rioter_enthusiasm
Units: Dimensionless
effect_on_enthusiasm = effect_of_rioters__on_enthusiasm.effect__of_cause_and_riot_on_enthusiasm
Units: Dimensionless
Initial_effect_of_Enthusiasm = 0.5
Units: Dimensionless
maximum_rioter_enthusiasm = 1
Units: Enthusiasm
natural_loss_rate_of_enthusiasm = 0.4
Units: Per Day
rate_of_losing__enthusiasm =
effect_of__arrests__on_enthusiasm.effect_of_arrests_on_loss_of_enthusiasm* additional_loss_rate_of_enthusiasm_at_maximum_arrest_effect + natural_loss_rate_of_enthusiasm
Units: Per Day
reaction_time_of_Rioter = 0.3
Units: Days

Media Interest:
Media__Interest(t) = Media__Interest(t - dt) + (increase__interest - lose__interest) * dt
INIT Media__Interest = Initial_Media_effect * maximum_media_interest
Units: Media
INFLOWS:
increase__interest = effect__on_Media*available__media_interest/reaction_time_of_Media_to_Events
Units: Media/Days
OUTFLOWS:
lose__interest = Media__Interest*rate_of_losing_Interest
Units: Media/Days
available__media_interest = (maximum_media_interest-Media__Interest)
Units: Media
Effect_of_media = Media__Interest/maximum_media_interest
Units: Dimensionless
effect__on_Media = effect_of_rioters__on_media.effect_on_media
Units: Dimensionless
Initial_Media_effect = 0
Units: Dimensionless
maximum_media_interest = 1
Units: Media
rate_of_losing_Interest = 0.7
Units: Per Day
reaction_time_of_Media_to_Events = 0.2
Units: Days

Population with Riot:
Arrested_Rioters(t) = Arrested_Rioters(t - dt) + (arrests__per_day) * dt
INIT Arrested_Rioters = 0
Units: People
INFLOWS:

arrests__per_day = Rioters*Arrest_Rate
Units: People/Days
Ex_Ex_Rioter(t) = Ex_Rioter(t - dt) + (leave_riot) * dt
INIT Ex_Rioter = 0
Units: People
INFLOWS:
leave_riot = Rioters/time_in__riot
Units: People/Days
Potential__Sympathisers(t) = Potential__Sympathisers(t - dt) + (-become__sympathetic) * dt
INIT Potential__Sympathisers = Total_Population-Initial_Sympathisers
Units: People
OUTFLOWS:
become__sympathetic = Potential__Sympathisers*fraction_of__population__influenced_per_day
Units: People/Days
Rioters(t) = Rioters(t - dt) + (join__riot - leave_riot - arrests__per_day) * dt
INIT Rioters = Initial_Rioters
Units: People
INFLOWS:
join__riot = Rioters*number_influenced_to_join_per_rioter
Units: People/Days
OUTFLOWS:
leave_riot = Rioters/time_in__riot
Units: People/Days
arrests__per_day = Rioters*Arrest_Rate
Units: People/Days
Sympathisers(t) = Sympathisers(t - dt) + (become__sympathetic - join__riot) * dt
INIT Sympathisers = Initial_Sympathisers-Initial_Rioters
Units: People
INFLOWS:
become__sympathetic = Potential__Sympathisers*fraction_of__population__influenced_per_day
Units: People/Days
OUTFLOWS:
join__riot = Rioters*number_influenced_to_join_per_rioter
Units: People/Days
actual_number__of_contacts_per__day_per_person =
fraction_of_population_sympathisers*Potential_Number_of_Contacts_per_day_per_person
Units: Per Day
Arrest_Rate = 0.49
Units: Per Day
fraction_of_population_sympathisers = Sympathisers/Free_Population
Units: Dimensionless
fraction_of__population__influenced_per_day = effect_of_media__on_sympathy__for_riot.fraction_of_population_influenced_per_day
Units: Per Day
Initial_Rioters = 90
Units: People
Initial_Sympathisers = 25000
Units: People
number_influenced_to_join_per_rioter = actual_number__of_contacts_per__day_per_person*probability_of_succesful_influence
Units: Per Day
Potential_Number_of_Contacts_per_day_per_person = 18.9
Units: Per Day
probability_of_succesful_influence = effect_of_enthusiasm__and_media_on__riot_recruitment.probability_of_succesful_influence
Units: Dimensionless
Riot_Participants = Ex_Rioter+Arrested_Rioters+Rioters
Units: People
time_in__riot = effect_of_enthusiasm_on_riot_retention.time_in_riot
Units: Days
Total_Population = 50000
Units: People
Free_Population = Sympathisers + Rioters + Potential__Sympathisers + Ex_Rioter
Units: People
