A General Model of Church Growth and Decline

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Abstract

An earlier model of church growth (Hayward 1999) is extended to include long-term effects due to births, deaths and reversion from the church. It is proposed that only a subset of the church, the enthusiasts, are involved in the recruitment process, and only for a limited period of time after their conversion. It is found that the church reaches equilibrium in its proportion of society according to the potential of these enthusiasts to reproduce themselves, and the losses from the church. If this reproduction potential is below a threshold that depends on losses, then extinction occurs. If it is above a higher threshold, then the church sees rapid revival growth. The model is applied to a number of church denominations to examine their prospects for survival or revival growth. Generally, declining churches do so because their reproduction potential is inadequate, rather than due to excessive losses.

Key Words: Church Growth, Population Models, Diffusion, Differential Equations, Epidemics, Religion.

1 Introduction

1.1 Church Growth

In the last thirty years considerable effort has been expended in attempting to understand how churches grow and decline (Hayward 1999, Inskeep 1993). Some of this work has been conducted by churches, and other religious groups, and includes both data gathering (e.g. Brierley 1997), and qualitative work (e.g. McGavran 1990), in order to enhance their prospects of growth. However there is also a growing body of sociological literature attempting to understand church growth, much of it in response to Kelley (1986) who claimed, in a book originally published in 1972, that strict churches are strong and hence grow, whereas liberal ones are weak and hence decline. As such there has developed a change of emphasis in the sociology of religion away from secularization theory, which assumed that religion always declines as society becomes more advanced, to a recognition of the robustness and growth of churches (Warner 1993). Thus there is now considerable academic

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interest in why churches grow (e.g. Fink & Stark 1992, Hoge & Roozen 1979, Iannaccone 1994a, Roozen & Hadaway 1993, Stark & Bainbridge 1985).

Theories of church growth tend to concentrate either on factors in society that enhance or inhibit growth, or factors within the churches themselves. For example do churches grow because they happen to be in areas of high birth rate or migration (Hoge 1979), or do they grow because they are strict, as proposed by Kelley (1986). Such theories are usually tested by checking if the factors correlate with church growth (e.g. Chaves 1989, Hadaway 1993, Hoge 1979, Iannaccone 1994b). However such theories do not provide quantitative causal links between the factors and growth.

In addition to these statistical approaches there are models that have been developed using rational choice theory, particularly the supply-side theories (Fink & Stark 1992, Iannaccone 1992). Although not all rational choice models have implications for growth, the supply-side theories do predict higher growth for smaller churches due to the need to compete more aggressively in the religious market place (Perl & Olson 2000). A similar model by Iannaccone, Olson & Stark (1995) was developed in order to provide the link between commitment and growth, as discussed by Kelley (1986). It predicts growth on the basis of religious resources such as time and money. However in none of these models is the growth described in terms of the fundamental causes: people joining and leaving the church. For this a dynamical model is required.

1.2 Dynamical Model of Church Growth

As a church is a subset of society, population models should provide a useful starting point for a dynamical model of church growth. One of the simplest models of population growth is the exponential model, which has been used by Stark to explain the rise of Christianity (Stark 1996a), and predict the future growth of the Mormon church (Stark 1996b). However no underlying reason was given as to why the growth should be exponential. The model was empirical rather than theoretical.

It has been noted that churches often recruit through friendship networks and personal contact (Hayward 1999, Olson 1989, Stark & Bainbridge 1985). Sargent (2000, p.112-116) notes that the highly successful "seeker churches" largely recruit through word of mouth. In addition Hadaway (1993) observes that churches that grow faster report more recruitment activity among their members. As such some sort of social diffusion model would appear a good candidate for a dynamical model of church growth.

Diffusion models are used extensively in the study of the spread of innovations and marketing (Kumar & Kumar 1992, Mahajan, Muller & Bass 1990), and usually assume the person adopting the innovation retains their influence over those who have yet to adopt it. However this will not necessarily be true in the case of church recruitment as members of the church can lose their network of unbelieving friends (Kelley 1986, Olson 1989), or run out of the "charisma" necessary to attract people (Poloma 1997). Church recruiters behave more like a person carrying a disease who only remains infected for a time. Thus Hayward (1999, 2000, 2002) developed a model of church growth, called the simple church growth model, or limited enthusiasm model, based on the general epidemic model of Kermack & McKendrick (1927).

The fundamental thesis of the simple church growth model is that church growth is driven by a subset of believers, the enthusiasts, who after a time lose their recruitment potential and become inactive. It follows that the church's growth follows a similar pattern to that of an epidemic, or the spread of a rumor (Coleman 1964). The model gave a good description of church growth in the short term, in particular the rapid growth often associated with religious revivals (Hayward 2002). Such

rapid growth was predicted to occur if the proportion of unbelievers exceeded a threshold (Hayward 1999, eq.23), ceasing before society is completely converted. The growth burned out because the enthusiasts' effort was increasingly ineffective in a shrinking pool of unbelievers. Similar rapid growth, threshold, and burnout effects are noted in other social situations where information is spread by word of mouth (Gladwell 2000). However the simple church growth model did not deal with the long-term growth of a church. It is the purpose of this paper to extend this model to include long-term effects such as births and deaths. The extended model will be referred to as the general church growth model.

There are other models that tackle long-term organizational growth. For example Sandell (2001) uses organizational ecology to develop models of the trade union movement and the church in Sweden, whereas Gaynor, Morrow & Georgiou (1991) use a system dynamics simulation to investigate the decline of a religious order. However neither of these models link growth to the behavior of individuals in the organizations. The former links the growth of the movement to that of its subgroups, e.g. congregations, making up that movement, and the latter to the desire for the organization to reach a target number. By contrast the general church growth model will retain the thesis that church growth is driven by enthusiasts.

There are also models of populations that use processes other than the simple diffusion/epidemic model. For example there are threshold models that take into account variations in the natural resistance of potential adopters (Granovetter & Soong 1983). People are deemed to have different thresholds of adoption which may depend on how many others have already adopted. This can lead to alternative models of diffusion (Braun 1995).

By contrast Baggs & Freedman (1990) model the growth of a bilingual population living among a unilingual one using a predator-prey model. The bilingual population acts as the predator. Like the diffusion models all the bilingual population are involved in the influence – there is no need for the concept of an enthusiast. Similarly Crane, Boccara & Higdon (2000) use a predator-prey model to analyze the growth of street gangs.

As this paper seeks to extend Hayward's (1999) model into a more general one, the epidemic approach will be retained in order to model the concept of enthusiast, the key feature of the church's growth. In addition the style of the general model will be kept deterministic. Although many types of models could be constructed using either stochastic or simulation methods (Bartholomew 1982, Gilbert & Troitzsch 1999, Hayward 2000), the deterministic approach is used here in order to extract principles and estimate parameters from data.

1.3 Overview of Paper

The main aims of this paper are:

1. To extend the simple church growth model to include the effects of reversion, births, deaths and converts who lack enthusiasm.

2. To extract principles that describe the long-term behavior of a church, given that its growth is driven by enthusiasts.

3. To use the model to investigate the current growth and decline of churches.

The model is constructed in section 2 with its analysis given in section 3. Only stationary and growing populations are considered as few countries have declining populations. A number of principles of church growth are extracted from the model in section 4, with the discussion kept largely free from mathematics to make it accessible to a non-mathematical audience. In section 5

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the model is applied to church denominations from the UK and USA, and to a growing world-wide religion.

2 Model Construction

2.1 Conversion and Limited Enthusiasm

The limited enthusiasm church growth model given in Hayward (1999) is based on three fundamental assumptions:

1. Unbelievers are converted, and recruited, into the church through contact with a subset of believers, called enthusiasts, or active believers.

2. After a period of time, the enthusiasts cease to be active in conversion, remaining in the church as inactive believers.

3. The enthusiastic period starts immediately after an unbeliever is converted.

The reasons why an enthusiast does not retain their conversion potential are explained in detail in Hayward (1999, 2002). Primarily, the new believer not only loses their enthusiasm to recruit after a period of time, but also loses their network of unbelieving friends as they become integrated into the life of the church. It is possible for existing believers to have their enthusiasm rekindled, but not so easy for them to establish a network of potential converts. Thus it is deemed that new believers are the primary means through which the initial contacts with unbelievers are made.

It therefore follows that the dynamics of the growing church resembles that of the spread of a disease, with the enthusiasts being the equivalent of those infected with the disease. The unbelievers are like the susceptibles and the inactive believers are like those who are no longer infected but remain immune to acquiring the disease again. The model is thus the same as the general epidemic model of Kermack & McKendrick (1927), (Hayward 1999).

In addition it will also be assumed that, unlike Hayward's (1999) original model, only a fraction of the new converts are enthusiasts. The remainder become inactive believers straight away and never play any part in the recruitment process, perhaps due to shyness, social isolation or because their conversion came about for social reasons only, lacking any deep conviction (Hayward 2002). If this fraction is unity, the original church growth model is reproduced.

The population is split into three categories, represented by the variables: U, the number of unbelievers; A, the number of enthusiasts (active believers); and B, the number of inactive believers; with $C = A + B$, the total number in the church.

Let τ_a be the average duration of the enthusiastic period, C_a be the number converted by one enthusiast during the whole of their enthusiastic period[1], and g be the fraction of converts who become enthusiasts. Then the growth of the church is given by the differential equations

$$\frac{dU}{dt} = -\frac{C_a}{\tau_a}A \tag{1}$$

$$\frac{dA}{dt} = g\frac{C_a}{\tau_a}A - \frac{A}{\tau_a} \tag{2}$$

$$\frac{dB}{dt} = (1-g)\frac{C_a}{\tau_a}A + \frac{A}{\tau_a} \tag{3}$$

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[1]In Hayward (1999) τ_a was called D, and C_a was called $n_i(t)$. $n_i(0)$ becomes the conversion potential C_p.

This model now differs from the general epidemic model of Kermack & McKendrick (1927) as not all infected people are themselves infectious.

The actual number of unbelievers converted will depend on the probability that an enthusiast makes contact with an unbeliever P_u. Thus $C_a = P_u C_p$ where C_p is the *conversion potential*: the number of converts one enthusiast would make if the whole of the population were unbelievers.

If the populations are homogeneously mixed, then P_u is given by the fraction of the unbelievers in the population $P_u = U/N$, where N is the total population, the sum of all the categories.

C_p is taken to be independent of τ_a as the exhaustion of the friendship network is the main reason for the enthusiastic period being curtailed. Only if the enthusiasm to recruit ceased long before the network was exhausted would the conversion potential be higher for a longer enthusiastic period.

C_p is also taken as independent of the total population size N, as the size of the network of friends is independent of N in large communities. There are only so many contacts a person can hold down. This is the fixed contacts model (Hayward 1999). In smaller communities the size of this network may depend on the population size, $C_p \propto N$, or crowd model (Hayward 1999). However as the new model is going to be applied to the church at the denominational level, or larger, the fixed contacts model is assumed. Thus the conversion potential of an enthusiast C_p is taken as a constant.

2.2 Adult Reversion

Although churches gain people through conversion, they can also lose them through adults leaving the church (children will be treated below). The reasons for such reversion are numerous, varying from loss of faith to moving house, with no one mechanism dominating (Ritcher & Francis 1998). There will be no attempt to model the detail of such processes here. Instead reversion is treated as proportional to the size of the church. In this model there will be separate reversion rates for enthusiasts and inactive believers, α_a and α_b, respectively.

It is further assumed that those who leave the church may not be immediately open to re-conversion (Brierley 2000, p.84), (Wilson & Sherkat 1994). Thus the leavers enter a new population category: the hardened unbelievers H. After a period of time the hardened unbelievers become open to re-conversion again, entering category U at a rate ϵ. Of course some will never be re-converted.

The re-conversion of leavers is a significant source of converts, according to surveys. For example Perrin, Kennedy & Miller (1997, Table 1) found that in some new evangelical denominations although 29% of those joining had no *immediate* affiliation to any church, only 13% had not had any past church involvement. Thus there is considerable recycling of church members, even when there is a gap after leaving.

2.3 Births and Deaths

An additional source of growth for the church comes from retaining the children of believing parents. This model assumes such children are counted in the church's numbers from birth. This is generally true for attendance figures but not for membership figures, which are delayed anything from around 8 years in Catholic churches to around 16 years in conservative evangelical ones. However given that homogeneous mixing of age groups will smooth out any such delays, it is assumed, as in many other population models (e.g. Baggs & Freedman 1990), that a fixed fraction of the children of believers are lost from the church each year. Figures for this loss vary from about a third to three quarters (Brierley 1991, p.87), (Johnson, Hoge & Luidens 1993).

Let the fraction of children of enthusiasts retained be f_{au}, with f_{bu} for those of inactive believers. It is assumed that the children of unbelievers, including the hardened, are born unbelievers. Although such children may attend a Sunday School, they rarely attend church without at least one parent.

For completeness it is assumed that only a fraction of the children of enthusiasts who remain in church also become active f_{aa}, whereas those of inactive believers are never active. Thus there is a small additional source of new enthusiasts: some of the children of enthusiasts.

The church will lose numbers through deaths. It is assumed that all categories of people have the same death rate d and the same birth rate b, noting that this may not be true for all denominations and religions.

2.4 Equations

From the above assumptions the equations (1 – 3) are extended to:

$$\frac{dU}{dt} = -\frac{C_p}{\tau_a N}UA + bU + bH - dU + \epsilon H + f_{bu}bB + f_{au}bA \tag{4}$$

$$\frac{dA}{dt} = \frac{gC_p}{\tau_a N}UA - \frac{1}{\tau_a}A - \alpha_a A - dA + (1 - f_{au})f_{aa}bA \tag{5}$$

$$\frac{dB}{dt} = \frac{(1-g)C_p}{\tau_a N}UA + \frac{1}{\tau_a}A - \alpha_b B - dB + (1 - f_{bu})bB$$
$$+ (1 - f_{au})(1 - f_{aa})bA \tag{6}$$

$$\frac{dH}{dt} = \alpha_a A + \alpha_b B - \epsilon H - dH \tag{7}$$

where $N = U + A + B + H$ is the total population, which is a function of time. These equations represent the general model of church growth and decline.

3 Model Analysis

3.1 Equilibria and Stability

Generally the total population N is changing: $\dot{N} = (b-d)N$, thus there is no equilibrium. However equilibrium is achieved in the *proportions* that each population group occupies, $\bar{U} = U/N$ etc., even in a growing population. Transforming (4 – 7) into the proportional variables gives:

$$\frac{d\bar{U}}{dt} = -\frac{C_p}{\tau_a}\bar{U}\bar{A} + b\bar{H} + \epsilon\bar{H} + f_{bu}b\bar{B} + f_{au}b\bar{A} \tag{8}$$

$$\frac{d\bar{A}}{dt} = \frac{gC_p}{\tau_a}\bar{U}\bar{A} - \frac{1}{\tau_a}\bar{A} - \alpha_a\bar{A} - b\bar{A} + (1 - f_{au})f_{aa}b\bar{A} \tag{9}$$

$$\frac{d\bar{B}}{dt} = \frac{(1-g)C_p}{\tau_a}\bar{U}\bar{A} + \frac{1}{\tau_a}\bar{A} - \alpha_b\bar{B} - f_{bu}b\bar{B}$$
$$+ (1 - f_{au})(1 - f_{aa})b\bar{A} \tag{10}$$

$$\frac{d\bar{H}}{dt} = \alpha_a\bar{A} + \alpha_b\bar{B} - \epsilon\bar{H} - b\bar{H} \tag{11}$$

Setting (8 – 11) to zero gives two equilibrium points as (9) is non-linear.

The trivial solution is $\bar{A} = 0$ giving an equilibrium point of $(1, 0, 0, 0)$. Thus the church's proportion of society tends to zero. In a growing population the church may survive and even grow but it fails to keep pace with the growth of the general population.

The non-trivial solution of (9) set to zero gives:

$$\bar{U} = \frac{1 + \tau_a \left(\alpha_a + b\hat{L} \right)}{R_p} = \bar{U}_{crit} \tag{12}$$

where

$$\hat{L} = 1 - f_{aa} + f_{aa}f_{au} \tag{13}$$

and R_p is the *reproduction potential*:

$$R_p = gC_p \tag{14}$$

From (5) R_p is the number of *enthusiasts* that would be made by one enthusiast during their enthusiastic period, given that the whole population are unbelievers. It is a measure of how fast enthusiasts reproduce themselves, and is the equivalent of R_0, the basic reproductive rate in epidemiology (Anderson & May 1987).

Substituting (12) into (10) and (11) gives:

$$\bar{B} = \hat{q}\bar{A} \tag{15}$$
$$\bar{H} = \left(\hat{p}_a + \hat{p}_b\hat{q} \right) \bar{A} \tag{16}$$

where

$$
\begin{aligned}
\hat{q} &= \frac{1 + \tau_a \left((1-g)\alpha_a + b\hat{L} - f_{au}bg \right)}{g\tau_a \left(\alpha_b + bf_{bu} \right)} \\
\hat{p}_a &= \frac{\alpha_a}{\epsilon + b} \\
\hat{p}_b &= \frac{\alpha_b}{\epsilon + b}
\end{aligned}
\tag{17}
$$

Using (12), (15) and (16) in $1 = \bar{U} + \bar{A} + \bar{B} + \bar{H}$ gives the remaining equilibrium values as

$$\bar{A} = \frac{1 - \bar{U}_{crit}}{\hat{M}} \tag{18}$$

$$\bar{B} = \frac{\hat{q} \left(1 - \bar{U}_{crit} \right)}{\hat{M}} \tag{19}$$

$$\bar{H} = \frac{\left(\hat{p}_a + \hat{p}_b\hat{q} \right) \left(1 - \bar{U}_{crit} \right)}{\hat{M}} \tag{20}$$

where

$$\hat{M} = 1 + \hat{q} \left(1 + \hat{p}_b \right) + \hat{p}_a \tag{21}$$

The Jacobian for $(8 - 11)$ is:

$$
J = \begin{bmatrix}
-\frac{C_p}{\tau_a}\bar{A} & -\frac{C_p}{\tau_a}\bar{U} + f_{au}b & f_{bu}b & b + \epsilon \\
\frac{gC_p}{\tau_a}\bar{A} & \frac{gC_p}{\tau_a}\left(\bar{U} - \bar{U}_{crit} \right) & 0 & 0 \\
\frac{(1-g)C_p}{\tau_a}\bar{A} & \frac{(1-g)C_p}{\tau_a}\bar{U} + \frac{gC_p}{\tau_a}\bar{U}_{crit} - bf_{au} - \alpha_a & -\alpha_b - f_{bu}b & 0 \\
0 & \alpha_a & \alpha_b & -\epsilon - b
\end{bmatrix}
$$

For the equilibrium point $(1,0,0,0)$ this has one zero and two negative eigenvalues. Thus stability is determined solely by the remaining eigenvalue:

$$\lambda = \frac{gC_p}{\tau_a}\left(1 - \bar{U}_{crit}\right) \tag{22}$$

so that $(1,0,0,0)$ is stable only when $\bar{U}_{crit} \geq 1$. Likewise it can be shown, after a lengthy calculation, that the non-trivial equilibrium point is stable when $(1,0,0,0)$ is unstable (and vice versa). Thus the church will achieve a stable proportion of society provided $\bar{U}_{crit} \leq 1$, otherwise its "market share" tends to zero.

From (18) and (19) the non-trivial equilibrium proportion of the church in the population is:

$$\bar{C} = \bar{A} + \bar{B} = \frac{(1+\hat{q})\left(1 - \bar{U}_{crit}\right)}{\hat{M}} = \frac{\left(1 - \bar{U}_{crit}\right)}{1 + (p_b\hat{q} + p_a)/(\hat{q}+1)} \tag{23}$$

If $\alpha_a = \alpha_b$ this simplifies to:

$$\bar{C} = \frac{\left(1 - \bar{U}_{crit}\right)}{1 + \hat{p}_a} \tag{24}$$

3.2   Enthusiasm of the Church

One measure of the enthusiasm, or life, of the church is the fraction of the church who are enthusiasts. At the non-trivial equilibrium this is computed from (15):

$$\frac{\bar{A}}{\bar{A} + \bar{B}} = \frac{A}{A + B} = \frac{1}{1 + \hat{q}} \tag{25}$$

where $\hat{q}$ is given by (17). It is independent of the critical value $\bar{U}_{crit}$, and thus independent of R_p.

3.3   Sensitivity of Parameters at Equilibrium

It is useful to compare the effects of different parameter changes on the equilibrium. In particular whether any parameter changes have a larger effect than changes in R_p. For simplicity consider the case where the two reversion rates are equal (24) and let $\alpha = \alpha_a = \alpha_b$. Small changes in the parameters will induce a small change in the equilibrium proportion in church:

$$\frac{\delta\bar{C}}{\bar{C}} = \frac{R_p}{\bar{C}}\frac{\partial\bar{C}}{\partial R_p}\frac{\delta R_p}{R_p} + \frac{\alpha}{\bar{C}}\frac{\partial\bar{C}}{\partial\alpha}\frac{\delta\alpha}{\alpha} + \frac{\tau_a}{\bar{C}}\frac{\partial\bar{C}}{\partial\tau_a}\frac{\delta\tau_a}{\tau_a} + \cdots \tag{26}$$

where $\delta R_p/R_p$ is the fractional (percentage) change in R_p etc. In order to compare the relative effects of identical percentage changes in any two parameters x, y define the relative sensitivity $S(x, y)$ as the ratio of the appropriate terms in (26). Thus comparing changes in R_p with those τ_a:

$$S(R_p\tau_a) = \left|\frac{R_p}{\tau_a}\frac{\partial\bar{C}/\partial R_p}{\partial\bar{C}/\partial\tau_a}\right| = 1 + \frac{1}{(\alpha + b\hat{L})\tau_a} \tag{27}$$

which is greater than unity. Likewise comparing R_p with the reversion rate:

$$S(R_p, \alpha) = \frac{\left(1 + \tau_a(\alpha + b\hat{L})\right)(1 + \hat{p})(\epsilon + b)}{\alpha\left(\tau_a(1 + \hat{p})(\epsilon + b) + (1 - \bar{U}_{crit})R_p\right)} \tag{28}$$

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For values of R_p near the critical loss threshold (12), this reduces to

$$S(R_p, \alpha) \approx \frac{1 + \tau_a(\alpha + b\hat{L})}{\alpha \tau_a} \quad (29)$$

Similar expressions can be defined for other parameters, however they have even less effect in comparison to R_p than the two above.

3.4 Equilibria With No Inactive Losses

When there are no losses from the inactive believers the equations $(8 - 11)$ give a line of equilibrium points. Let $\alpha_b = f_{bu} = 0$ then from (17) and (21) both $\hat{q}$ and $\hat{M}$ are zero. Thus both $\bar{A}$ and $\bar{H}$ are also zero. As in the Kermack-McKendrick case all the variables are then in equilibrium, regardless of the value of $\bar{U}$. Thus growth has stopped because there are no more enthusiasts, and the church survives purely with inactive believers on the equilibrium line $(\bar{U}, 0, 1 - \bar{U}, 0)$, where $\bar{U} \leq \bar{U}_{crit}$.

3.5 Small Church Approximation

Small churches recruit from an unbelieving pool much bigger than themselves. Thus, unless reversion is very large and ϵ very small, then $U \gg A, B, H$, so that $U \approx N$. Thus (9) becomes the exponential model for the enthusiasts:

$$\frac{d\bar{A}}{dt} = \left(\frac{R_p}{\tau_a} - \frac{1}{\tau_a} - \alpha_a - b + (1 - f_{au}) f_{aa} b \right) \bar{A}$$

which gives exponential growth, $\bar{A} = \bar{A}_0 e^{\theta t}$, with a rate of

$$\theta = \frac{R_p}{\tau_a} - \frac{1}{\tau_a} - \left(\alpha_a + b\hat{L} \right) \quad (30)$$

The church $\bar{C}$ also grows at the same exponential rate, apart from a transient. Adding together (9) and (10) gives:

$$\frac{d\bar{C}}{dt} = \frac{C_p}{\tau_a} \bar{A} - \alpha_a \bar{A} - \alpha_b \bar{B} - f_{au} b \bar{A} - f_{bu} b \bar{B}$$

If the loss rates for both types of believers are the same this becomes

$$\frac{d\bar{C}}{dt} + (\alpha_b + f_{ab}b) \bar{C} = \frac{C_p}{\tau_a} \bar{A}_0 e^{\theta t}$$

whose only growing solution is at the rate (30).

3.6 Critical Loss and Extinction Thresholds

From the equilibrium analysis a critical loss threshold can be defined and compared with R_p (14). Using (12), the critical loss threshold is:

$$R_{crit} = 1 + b\hat{L}\tau_a + \alpha_a \tau_a \quad (31)$$

If the reproduction potential is above this threshold, the church will stabilize to a fixed proportion of society $R_p > R_{crit}$. Otherwise its proportion vanishes, although the church itself doesn't necessarily become extinct.

In addition there is an extinction threshold $R_{extinct}$, which will be lower than the critical threshold in a growing population $(b > d)$. To obtain this threshold transform the variable U alone to its proportion so that the variables of the system are $(\bar{U}, A, B, H)$. Then (4) becomes:

$$\frac{d\bar{U}}{dt} = \left(-\frac{C_p}{\tau_a}\bar{U}A + bH + \epsilon H + f_{bu}bB + f_{au}bA \right) \frac{1 - \bar{U}}{A + B + C} \tag{32}$$

where N has been replaced by $(A+B+C)/(1-\bar{U})$. The other variables have equations similar to (5 – 7) with U replaced by $\bar{U}$. These equations have an equilibrium at $(\bar{U} = 1, A = 0, B = 0, H = 0)$, the extinction of the church.

Calculating the Jacobian of (32) with (5 – 7), and applying the extinction equilibrium point, gives only one eigenvalue that could be positive:

$$\lambda = \frac{gC_p}{\tau_a} \left(1 - \bar{U}_{crit} \right) + (b - d) \tag{33}$$

Thus extinction occurs when this eigenvalue is negative. Combining (12), (31) and (33) gives the extinction threshold as:

$$R_{extinct} = R_{crit} - \tau_a (b - d) \tag{34}$$

In a stationary population $(b = d)$ the two thresholds are the same.

For a church to avoid extinction the reproduction potential must exceed the extinction threshold $R_p > R_{extinct}$, otherwise the enthusiasts, the source of the church's growth, will die out.

3.7 Revival Growth Thresholds

The reproduction potential R_p governs the rate at which enthusiasts reproduce themselves. The number of enthusiasts will increase if R_p exceeds a threshold, leading to rapid growth in the numbers being converted into the church. Because this pattern of growth is typical of religious revivals it is referred to as a revival growth threshold, and is the extension of the epidemic threshold of Kermack & McKendrick (1927), (Bailey 1975, p.82) (Hayward 1999, eq.12). However there is a higher threshold over which the *proportion* of enthusiasts rises. To distinguish the two revival growth thresholds the lower one is called the weak revival growth threshold – just growth in the enthusiasts' numbers – and the higher threshold is called the strong revival growth threshold.

Setting (5) equal to zero and solving for R_p, the weak revival growth threshold is:

$$R_{weak}(\bar{U}) = \frac{R_{extinct}}{\bar{U}} \tag{35}$$

Likewise (9) gives the strong revival growth threshold:

$$R_{revival}(\bar{U}) = \frac{R_{crit}}{\bar{U}} \tag{36}$$

Both are functions of the proportion of unbelievers in society. As the church grows the thresholds get lower, the proportion of enthusiasts decline, and then their numbers decline. Church growth slows as $R_{revival} \to R_p$, and eventually the church's proportion of society will stabilize to the non-trivial equilibrium value given by (23).

Figures 1 and 2 show the growth of a church with R_p above the extinction and critical thresholds. As such it survives. As R_p is also above the revival thresholds there is an increase in the proportion of enthusiasts (figure 1) which is reversed as the revival threshold rises above R_p (figure 2). The church eventually settles to its equilibrium, $R_p = R_{revival}$.
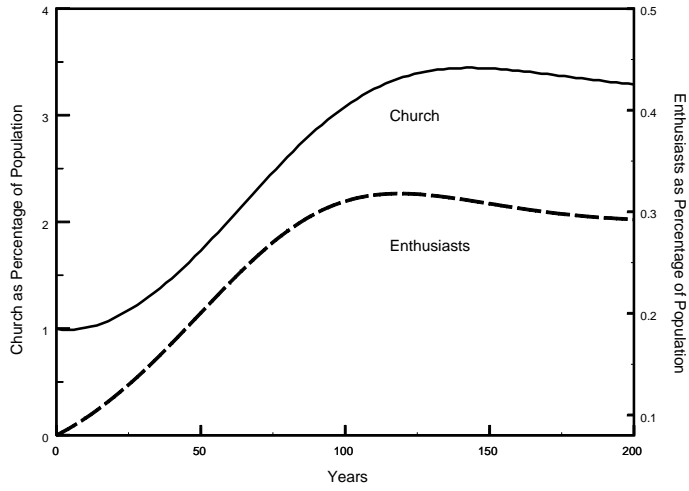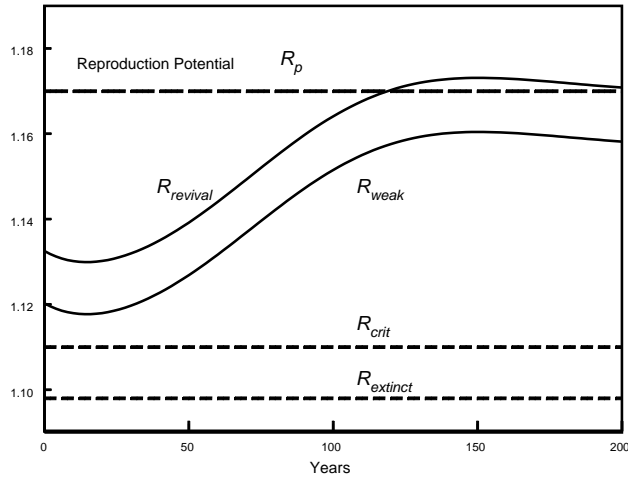
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Figure 1: Growth of church & enthusiasts

Figure 2: Reproduction potential compared with thresholds

4 Results

4.1 Equilibrium

A church's proportion of society will tend to an equilibrium. If the population is stationary then this equilibrium occurs in the church's numbers.

1. **Growth is limited by the reproduction potential**. Even with no reversion or child losses the growth of the church is limited. It is the limited enthusiasm of the enthusiasts

that prevents the church from converting the whole of society. In this case the equations (8 – 11) collapse to the Kermack-McKendrick model. The equilibrium becomes a line so that the final proportion in the church depends on the initial values (section 3.4). This final value is higher than the revival threshold value $\bar{C} = 1 - 1/R_p$, (23), but less than 1, the total population (Hayward 1999, section 3.3). Church growth has run out for a lack of enthusiasts, whose reproduction potential has not been sufficient to reproduce themselves once the revival threshold is exceeded, and the fraction of potential converts is dwindling.

If the child losses and reversion are non-zero then the final proportion in the church becomes even lower, and settles on the non-trivial equilibrium value (23). In this case the enthusiasts no longer become zero but reach the equilibrium value (18). The source of the new enthusiasts is enhanced by believers who have reverted back to unbelief and become potential converts again.

Thus it is incorrect to say that reversion and child loss alone prevent a church from growing. Churches stop growing because they reach the limit at which their enthusiasts can sustain themselves given their reproduction potential and the size of the unbelieving pool. The failure to reproduce enthusiasts at a sufficient rate will limit growth.

2. **Equilibrium is independent of initial numbers, as long as there is inactive reversion**. The non-trivial equilibrium is a single point as long as either $\alpha_b > 0$ or $f_{bu} > 0$ (see section 3.4) thus, if stable, then all phase paths end up at this point regardless of initial values. This means that the limit achieved by a church will depend solely on its parameters, such as its reproduction potential, not on its size or its number of enthusiasts. A small church will achieve the same equilibrium in society as a large one if it has the same parameters. Improvement in growth prospects can only be achieved through stemming losses or increasing the reproduction potential.

By contrast, if a church has no losses from the inactive believers then there is a line of equilibrium points, and the limit to the church's growth does depend on the initial values. Such a church will remain at that equilibrium, be it large or small, even if the reproduction potential is reduced or the enthusiasts become extinct. However if it then suffers losses, it will collapse to a single stable equilibrium, either zero or non-zero depending on the parameters.

3. **Changing the conversion potential without changing the reproduction potential has little effect on the long-term growth of a church**. If a church is able to increase its recruitment, but unable to raise its reproduction potential (higher C_p at the expense of lower g), the limit of the church's growth changes very little. This follows from (23) with typical reversion rates of less than 10% and an enthusiastic period of around 2 years. As such $\hat{q}$, the only variable affected by g independently of R_p, is large and factors out. In the case where the reversion rates are equal the equilibrium is completely unaffected by g or C_p separately (24). Thus, although the short-term growth of the church will be enhanced through recruiting inactive believers, the long-term growth is unaffected as the reproduction of enthusiasts is unchanged.

Likewise the enthusiasm of the church at equilibrium (25) cannot be improved by increasing the conversion potential alone. The equilibrium fraction of enthusiasts is independent of the conversion potential. The enthusiasm is only improved by extending the enthusiastic period, stemming losses, or increasing the fraction made enthusiastic.

4. **Changes in the reproduction potential have more effect on church numbers than changes in other parameters**. This follows from (27) and (28). With typical reversion rates and enthusiastic periods both sensitivities are considerably bigger than 1. Thus, if a church is to enhance its long-term growth, increasing the reproduction potential will have more impact than stemming losses. This is particularly true near the critical loss threshold where changes in the reproduction potential have around ten times more impact than changes in the reversion rate (29).

4.2 Survival of Churches

A church will survive if its reproduction potential is above the extinction threshold (34), and tend to a finite proportion of society if it is above the critical threshold (31).

5. **A church needs to make enthusiasts not just converts if it is to avoid extinction**. This follows from section 3.6. If a church is heading for extinction then making converts alone will only give short-term growth. In the long term conversions dry up for a lack of enthusiasts and extinction follows (figure 3). The need to make enthusiasts, or contagious believers, in order to survive and grow is a recognized strategy in many fast growing churches (Miller 1997, Sargent 2000, Towns 1990).
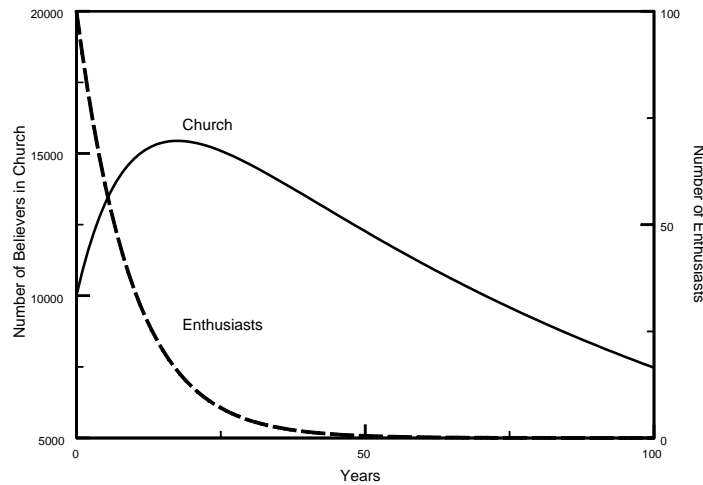

Figure 3: Church with high conversion rate but low reproduction potential

6. **In a growing population a church may survive, but lose its market share**. This happens when $R_{extinct} < R_p < R_{crit}$. Although the church grows, it fails to keep pace with the growth of the population and its proportion of society declines to zero. To make inroads into society R_p must exceed the critical threshold. Even without losses this must be bigger than 1, one enthusiast needing to make at least one new enthusiast. With losses etc. R_{crit} typically ranges from 1.05 to 1.15.

It follows that it is easier for a church to avoid extinction in a growing population. Without losses the extinction threshold is less than 1 (34), typically 0.98 for the USA and 0.93 for

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rapidly growing African countries.

7. **If the reproduction potential is inadequate then stemming losses will not avoid extinction**. Even without losses the extinction threshold has a minimum value. Often great emphasis is placed on retaining adults and children in order to turn a church around. However if its reproduction potential is too low then the church's inability to reproduce enthusiasts, not its losses, is at the heart of its decline.

8. **The survival of a church with a low reproduction potential is very sensitive to losses**. The one exception to result 7 is when the church loses no adults or children and survives on biological growth alone. In this case the equilibrium is a line and the church survives at whatever level it has reached, even if the reproduction potential is well under the extinction threshold (result 2). However if it *then* loses people, its decline becomes catastrophic. This is the Western European experience of the Christian church which had survived on generational retention alone with insufficient conversions. With the post-war drop in retention rates its inadequate reproduction potential has been exposed through its decline (Brown 2001, Ch.8)(Stark & Iannaccone 1994).

4.3 Revival Growth

Revival growth will take place if the reproduction exceeds the weak revival threshold (35). In addition if the strong revival growth threshold (36) is exceeded there will be revival growth in the proportion of the church in society. In these cases the church's growth follows the pattern of an epidemic as noted by a number of authors (Gladwell 2000, Greig 1998).

9. **A church needs to make enthusiasts not just converts to see revival growth**. It is the growth of enthusiasts, not believers, that drives revival growth (section 3.7). If a church initially consisted of all enthusiasts, but their reproduction potential was below the revival thresholds, then the church would grow for a time but it would not be revival growth. The enthusiasts would be declining and eventually the church would follow a similar pattern to figure 3. The church had a temporary growth in numbers but a decline in enthusiasm, or life, as measured by (25).

   Conversely the church may be declining but as long as the number of enthusiasts is increasing then the church will start growing again with revival growth. It is the growth in the enthusiasm of the church rather than numbers that is an indicator of its health.

10. **Revival growth depends on the proportion of unbelievers in society**. Both revival thresholds are inversely proportional to the number of unbelievers. As the church grows the unbelieving pool shrinks and the revival thresholds rise. Once they have risen above the reproduction potential the revival growth (in numbers or proportions) ceases and growth slows until equilibrium is reached (figures 1 & 2).

    It follows that revival growth is easier when a church is a small proportion of society as its revival threshold is lower. Thus revival growth is easier for a small sect to achieve compared with a substantial established denomination. Likewise revival growth is easier to achieve in Western European countries now than it would have been a hundred years ago when the church was larger.

11. **Losses make revival growth harder**. The revival thresholds depend on losses. The more people leave the church the higher the threshold. For example without any losses the revival growth threshold (36) for a church of 10% of the population is $1/\bar{U} \approx 1/0.9 \approx 1.1$. If reversion of 10% occurs with an enthusiastic period of 2 years then this threshold rises to 1.3, and a larger reproduction potential is required for revival growth.

12. **Revival growth in church numbers is easier in a growing population**. That is, when the birth rate exceeds the death rate then $R_{revival} > R_{weak}$, the latter being reduced by a factor depending on the population's growth rate (34, 35). Thus for rapidly growing African countries the weak revival threshold for a church of 10% is reduced from 1.1 to 1.03. Thus churches with even moderate reproduction potentials can see revival growth in numbers even though there is no such growth in their proportion of society.

    It is possible that the weak revival threshold could fall below the critical threshold. For the African example above, a church comprising 1% of the population has a weak threshold of 0.94, below the critical threshold of 1. If the reproduction potential lay between these two values then the church would see revival growth in its numbers, but it would fail to keep pace with the growing population with its market share tending to zero.

14. **Small growing churches grow exponentially**. This follows from the unbelieving pool being so large (section 3.5). As long as the church remains small, its revival growth can be given as a doubling time. Many small churches have growth rates of this pattern (Stark 1996b, Stark & Iannaccone 1997). If the exponential growth rate θ is known then an estimate can be obtained for the reproduction potential, given that τ_a and the loss rates are known from other sources. Thus, even from the exponential phase, it may be possible to estimate the limit of a church's growth.

5 Application to Church Data

5.1 General Considerations

Given suitable data it should be possible to provide estimates of the parameters in the model for church groups. Such data fitting will not prove the model correct, however if correct then it will provide an explanation for the church's growth or decline in terms of the fundamental assumption of the model – growth through enthusiasts.

Fitting data to dynamical systems where the analytical solution is unknown is not always straightforward (Schittkowski 2002, Scitovski & Meler 2002). However with the general smoothness of church data integrating with a Runge-Kutta-Fehlberg order 5/6 method, and optimizing using non-linear least squares (NLS), gives acceptable results for most churches. It is not possible to provide confidence limits for the parameters due to the lack of independency of the data points, or any statistical model for their errors (Schittkowski 2002). Following Jacobsen, Bronsen & Vekstein (1990) a heuristic measure is used to check the acceptability of parameter values. In this paper a percentage deviation at each data point is used.

The question as to what data to use is more problematic. Although many churches keep good membership data, the definition of membership can vary considerably. In strict churches it is usually a measure of clear commitment. e.g. those who regularly tithe. In broader churches it can include all who are deemed to be part of its religious community regardless of commitment. Thus,

in most cases, membership is a poor measure of attendance at the time. As such there may be a phase lag, or lead, between attendance and membership, depending on the denomination.

However provided the definition of membership for a given denomination has not changed during the period in question it should still give an indication of the change of attendance. Thus membership data can provide estimates of the parameters, even if not strictly reflecting the period in question. Membership figures will only be used if attendance figures are unavailable or unreliable.

No church denomination exists in isolation. The Christian church has many such denominations who both compete and cooperate with each other. For simplicity it will be assumed that for any such denomination the "unbelieving" pool U includes the other denominations, as well as other religions or those with no such attachment. This can be justified for the late twentieth century as denominational switching has become acceptable (Miller 1998, Perrin et al. 1997). In earlier centuries the unbelieving pool may have to exclude other denominations.

To avoid overparameterization only four parameters will be fitted from the data: R_p, τ_a, α and the initial fraction of the church that is enthusiastic A_0/C_0. It is assumed that the two reversion rates are the same $\alpha_a = \alpha_b = \alpha$. f_{aa}, f_{au}, f_{bu} and ϵ are estimated from other sources. The initial fraction of unbelievers that are hardened is chosen to reflect the general changes in the data variables. Different values of g, the fraction of converts who become enthusiastic, were used giving largely the same results for the chosen parameters. Thus it has not been possible to determine g or C_p separately.

5.2 UK Churches

Three church attendance surveys have been conducted for England in recent years by the same organization, Christian Research, using the same methodology. The data for selected denominations is given in table 1 (Brierley 1999, table 12.3.2). Although this only gives three data points it is suf-

Year	1979	1989	1998
All Churches	5,441,000	4,742,800	3,714,700
Anglican	1,671,000	1,266,300	980,600
Methodist	621,000	512,300	379,700
Catholic	1,991,000	1,715,900	1,230,100
Baptist	290,000	270,900	277,600
New Churches	64,000	167,000	230,500

Table 1: English Church Attendance Surveys - Christian Research

ficient to provide reasonable estimates of the reproduction potential of the different denominations. Most recent surveys suggest child loss in England of 50% (Brierley 2000, p.100). The softening rate, ϵ, is roughly estimated at a low value, 0.05, as only 20% of those who leave church return again, and that after an average of 10 years away (Brierley 2000, p.84).

The "new churches" are independent churches that became visible in the late 1970's (Walker 1989). They are similar to the "new paradigm" churches in the USA (Miller 1997). Their attendance survey figures were not used as these churches are not consistently defined over the period. Instead membership figures for the 1990's are used (Brierley 1999, 2001), as these churches generally estimate their membership by attendance.

The results for the least squares optimization are given in table 2.

Denomination	R_p	τ_a	α	$\frac{A_0}{C_0}$	Condition
Anglican	1.08	2.4	4%	2%	Extinction
Methodist	0.75	2.4	3.5%	6%	Extinction
Catholic	1.025	2.1	3%	6%	Extinction
Baptist	1.15	2.05	2.5%	1.6%	Slow Revival
New Churches	1.275	2.6	4%	11%	Revival

Table 2: Best Fit Parameters for English Denominations

The Anglican churches, mainly the Church of England, are below the extinction threshold. With R_p just over 1 it would avoid extinction if its losses were stemmed. The Catholic churches, largely the Roman Catholic, are in a similar position. The Methodist church has an even lower reproduction potential so that extinction cannot be avoided by stemming losses (result 7). They are not reproducing enough enthusiasts to survive (result 5). This appears to provide support for the secularization hypothesis in the UK (Brown 2001, Bruce 2001), rather than the supply-side models (Stark 1999). However Bruce's (2001) indication of extinction by 2030 may be unduly pessimistic as some reproduction is taking place. Extinction can be a very slow process.

On the other hand the Baptists have R_p over the revival growth threshold, but with a low fraction of enthusiasts. Although they had been in decline there is evidence that the number of enthusiasts was rising leading to moderate revival growth (result 9). It would take further data to confirm this.

In addition the new churches, with much higher R_p, are seeing significant revival growth. If their growth continues, they will be larger than the Methodist church by 2010, and larger than the Church of England by 2025, indicating a significant shift of religious commitment taking place in the UK. Although their numbers are not enough to counteract the decline in the larger denominations, if their reproduction potential stays the same then Christianity in the UK will grow again. As the Christian church would then be in the hands of a larger variety of smaller denominations this would lend some support to the supply-side theory, which predicts higher growth where there is greater competition (Stark & Iannaccone 1994).

It is interesting to note that there is little difference between the denominations in either enthusiastic period or reversion rate. Those that are growing do so because they are more effective at reproducing enthusiasts. Those that are declining do so because they are failing to convert people, rather than losing them.

Data were also fitted to the model insisting on minimum percentage errors at each point. Although a small range of values for R_p and α were generated the condition of the different denominations was unaffected.

5.3   Combating Church Decline

How could a declining church, such as the Church of England, turn itself into a growing one? Consider three policies that could be implemented in 2005 for that church:

1. halving all losses;

2. increasing the reproduction potential by a small amount (from 1.08 to 1.15 the Baptist figure);

3. both policies 1 and 2 together.

The results for the total church numbers are given in figure 4. The projected numbers for the Church of England in 2100 are less than 10% of its current value. Stemming the losses alone (policy 1) avoids extinction, and slows the decline. Eventually the church starts to grow in line with the population, with an equilibrium of 0.6% of society. At almost a third of the current value this policy has not turned the church around.
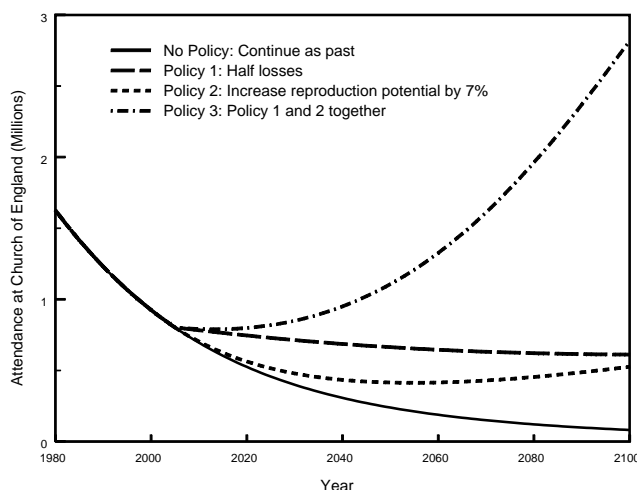

Figure 4: Policies to combat decline in Church of England attendance

Increasing the reproduction potential (policy 2) has less effect in the short term than policy 1, but it gets the church into a similar position by the end of the century. However as the number of enthusiasts is increasing policy 2 has a greater long-term effect than policy 1 with the equilibrium at 1.1% (an example of result 4). Although better than policy 1 its full impact is not achieved over a realistic time-span.

If both policies are applied together the result is dramatic, with significant revival growth taking place (figure 4). Reducing losses has made revival growth easier (result 11). An equilibrium value of 4.4% would eventually be achieved, with the church almost back to its 1900 value by 2200. Despite this it is still the early 2020's before the church recovers back to its 2005 value. Revival growth is slow in starting due to the small number of enthusiasts (Hayward 1999, section 5.4). However, as an unknown, but significant, minority of Church of England congregations have a similar ethos to the "new" churches then it is not inconceivable that such a change in growth pattern is already taking place.

5.4   USA Churches

Consistent attendance data for USA churches is difficult to obtain. Instead membership data is used as an estimate of attendance for the period 1975–1995 (Brierley 1997). The best fit parameters are in table 3.

Both the Episcopal and Methodist Churches are under the critical loss threshold, declining to a vanishing fraction of society (result 6). Although their optima are just over the extinction threshold the range of acceptable values includes extinction. Nevertheless, despite their obvious decline, they

Denomination	R_p	τ_a	α	$\frac{A_0}{C_0}$	Condition
Episcopal	1.1	1.8	6%	7%	Critical
United Methodist	1.12	3.2	3%	3%	Critical
Catholic	1.24	1.2	1%	1%	Weak revival
Southern Baptist	1.2	3.1	5%	8%	Revival ending
Pentecostal	1.25	2	6%	6%	Revival

Table 3: Best Fit Parameters for USA Denominations

are reproducing enthusiasts better than their counterparts in the UK. This lends some support to the claim that the USA is less prone to secularization than the UK (Bruce 2001, Stark 1999), as even the weaker churches are more robust.

Remarkably, the Catholic church is seeing weak revival growth in membership. However it is not clear if these membership figures accurately measure attendance. Also, the figures may be skewed by immigration, not taken into account in the general church growth model. Nevertheless the church appears far more robust than in the UK, where membership figures are falling.

The Pentecostal churches, which include the new paradigm churches (Miller 1997, Towns 1990), are seeing significant revival growth. However this revival appears nearer the end than the equivalent "new" churches in the UK. This is probably due to the USA figures containing a large number of traditional Pentecostal churches which are not growing as fast, hiding the true growth of the new paradigm churches.

Data for the Southern Baptists proved difficult to fit giving a wide variation in parameters whatever period was chosen. The optimum in table 3 was chosen for having reversion rates similar to other churches. These values show that they are nearing the end of a period of revival growth, and growing only in line with the USA population. Essentially, they have reached their market share, as determined by their ability to reproduce enthusiasts (result 10).

The bets fit parameters of table 3 are very much a first attempt. A more accurate analysis for USA churches could be obtained from consistently measured attendance figures, and if some of the parameters, such as reversion, could be derived from other sources.

5.5 Latter-Day Saints

One of the fastest growing religious groups in the world is the Church of Jesus Christ of Latter-Day Saints (LDS), or Mormon church. Consequently it has become a standard study for church growth. Stark (1984) has noted that its exponential growth is such that it will become one of the major world faiths during the 21st century with around 85 million members in 2080 (Stark 1996b). However it has also been noted by Anderson (2000) that this exponential growth cannot continue indefinitely. Instead he fitted the LDS membership data to a logistic model and found evidence for a slow down in growth with a carrying capacity of just over 30 million members.

The LDS are an ideal candidate for the general church growth model as much of their growth comes through short-term missionary work (Hepworth 1999). The missionaries are the enthusiasts of the model. Using the same membership data as Stark (1996b), which is a measure of those baptized as Mormons rather than those who currently attend, a growth rate of $\theta = 4.2\%$ from 1970-1995 is obtained. Thus, with similar reversion rates to other churches, and applying the small church approximation (result 14), (30) gives $R_p \approx 1.2$, giving an equilibrium value for the LDS of 4.5% of the world population.

According to this model the 2080 figure for the LDS church is predicted to be about 200 million, much higher than Stark's (1996b) estimate of 85 million. This is due to the assumption that all the world is part of the susceptible pool, a highly unrealistic assumption as not all countries are open to the spread of a new religion. If only a quarter of the world is open then the 2080 estimate drops to 60 million. If Anderson's (2000) low estimates are correct then it may be that the LDS are only contacting a fraction of the world's population.

It is interesting to note that the reproduction potential for the LDS is not radically different from some of the growing denominations in the UK and USA. The reason the LDS growth is larger is that they are applying this growth over many countries, giving a much larger pool of susceptibles (result 10).

5.6 Empirical Adequacy of the Model

Proving the correctness of dynamical models of a sociological nature is a controversial topic beyond the scope of this paper (Jacobsen et al. 1990). To prove that this model is adequate it will be regarded as sufficient to demonstrate the existence of enthusiasts within the church who alone are responsible for its growth. To test this hypothesis the opposite model was considered where *all* the believers were enthusiasts, effectively removing the category B. In most cases there was no acceptable fit to attendance or membership data, with the NLS optimum having some data points with very high percentage errors. Thus not all the believers are enthusiasts. This is reflected in the general church growth model giving very small durations for the enthusiastic period.

However there is also less quantitative evidence for enthusiasts. Enthusiasts are the high recruiters in the church but inevitably they also have enthusiasm for other aspects of the faith. This spills over into attendance at specialist meetings and the emergence of new forms of church life.

For example, in the UK, the last decades of the twentieth century have seen a dramatic rise in the number of annual Christian conventions and teaching weeks such as Spring Harvest, where demand outstrips supply. Most of these conventions are dominated by the newer churches where, according to the model, the number and quality of enthusiasts are higher. Thus there appears a category of believer that is willing to spend much more time on religious activities than just weekly church attendance. Some of these believers are good candidates for enthusiasts.

In addition there has been a large rise in new forms of worship and mid-week home groups (Miller 1998). Such changes in church life usually require a greater commitment from church members. Kelley (1986) argues that greater commitment correlates with missionary zeal, which would lead to a higher than normal reproduction potential. Again it is the churches with the high reproduction potentials that have these features of church life as the norm.

The most significant indicator of enthusiasts has been the rise of the Alpha course. Originating in a British Anglican church, albeit one with a new paradigm flavor, this course has spread to most countries of the world, and is used in numerous Christian denominations (Hayward 2002). The course is essentially evangelistic with people being invited on to the course by their believing friends. Those who are converted in turn invite their unbelieving friends onto the next course. Such people are candidates for enthusiasts. The rise in the number of registered courses, from 5 in 1992 to over 25,000 worldwide in 2003, indicates a significant rise in the number of enthusiasts in the church. Again it is the "new" churches who are the greatest users of the course. As yet the course shows no sign of burning out or of reaching an equilibrium.

6 Conclusion

6.1 General Conclusions

This paper extended the simple church growth model of Hayward (1999), extracted principles that govern a church's long-term growth, and applied the model to current church attendance and membership figures.

It was found that the proportion of society belonging to the church reaches an equilibrium as long as the reproduction potential of the enthusiasts is above a critical threshold determined by the other parameters. This equilibrium falls short of the whole population even when there are no losses. Growth is limited by the reproduction of enthusiasts. Generally the equilibrium is independent of the initial conditions, as long as there are some losses, and is more sensitive to the reproduction potential than the other parameters.

For a church to survive its reproduction potential must be higher than an extinction threshold. If that potential is inadequate then stemming losses alone will not help the church recover. On the other hand if the reproduction potential is above one, or both, of the revival thresholds, then it may see rapid growth in its numbers or even in its proportion of society. However as the church's share of society goes up that revival growth becomes harder to maintain. Although reducing losses may help in the short term, increasing the efficiency of enthusiasts will be the only way to continue such growth in the long term.

Thus the key to a church's growth, or survival, is how well the enthusiasts reproduce themselves. It is not sufficient for them to make converts; the converts must be enthusiasts also, active in the conversion of others. Generally the growing churches in the USA and the UK are growing because they have a higher reproduction potential than the declining ones. According to this model, the current decline in mainstream western Christian denominations is attributed to them having enthusiasts of insufficient quality to drive their growth rather than any losses they suffer. If this model is correct they must improve their reproduction potential to survive or recover.

6.2 Further Work

There are a number of possible extensions to the general church growth model:

1. As noted in the construction of the model it is possible to make inactive believers enthusiastic again, as happens in periods of revival (Greig 1998, Hayward 2002, Poloma 1997). However it is not so easy for them to restore a network of potential converts, thus some form of differentiation between new and renewed enthusiasts may be required. This would be a major extension of the current model.

2. Churches do not exist in isolation but compete for their market share of potential converts, as well as recruiting from each other's ranks (Fink & Stark 1992, Perl & Olson 2000, Perrin et al. 1997). This has been handled in the current model by assuming that all other churches are unbelievers. A model could be developed where the interaction between different churches is separate from those between church and society. Such a model runs the danger of having too many parameters, especially when it is realized just how many denominations and religions there are in some societies. The problem is made worse as many churches increasingly co-operate, and attach less importance to denominational identity (Miller 1997, Sargent 2000). Such an extension is beyond the scope of this paper.

3. The conversion potential is not necessarily constant. It may be higher for churches with a small market share, as suggested in the supply-side theories (Stark & Iannaccone 1994). However it could be lower for small churches who struggle to make their beliefs convincing, as predicted in the social isolation model (Perl & Olson 2000). Indeed the conversion potential may even depend on the strictness or purity of the church if Kelley's (1986) thesis is correct.

The model will be extended in future publications.

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Glossary

Dynamical variables:

t Time
U Number of unbelievers open to conversion
A Number of enthusiasts (active believers)
B Number of inactive believers
H Number unbelievers hardened against conversion
N Population number
C Total number in church $= A + B$
$\bar{U}$ Bar indicates proportion of population U/N

Parameters:

C_p	Conversion potential: the number of converts one enthusiast would make if the whole of the population were unbelievers
g	Fraction of converts who become enthusiasts
R_p	Reproduction potential: the number of enthusiasts one enthusiast would make if the whole of the population were unbelievers (14)
τ_a	Duration of enthusiastic or active period
α_a	Adult reversion from enthusiasts
α_b	Adult reversion from inactive believers
ϵ	Rate at which hardened unbelievers become open to conversion
f_{au}	Fraction of enthusiasts' children who become unbelievers
f_{bu}	Fraction of inactive believers' children who become unbelievers
f_{aa}	Fraction of enthusiasts' children who, if believers, become enthusiasts

Thresholds:

$R_{extinct}$	Extinction (34)
R_{crit}	Critical loss (31)
R_{weak}	Weak revival growth (35)
$R_{revival}$	Strong revival growth (36)